COMPUTER - AIDED DESIGN AND ANALYSIS OF GEAR - BOX SHAFTS

By

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APRIL, 1985

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A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By
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to the
DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
APRIL, 1985

12 JUN 1985

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Submitted on 18/4/85

CERTIFICATE

This is to certify that the work contained in this thesis, entitled 'COMPUTER-AIDED DESIGN AND ANALYSIS OF GEAR-BOX SHAFTS' has been carried out by Shrinivas. V. Umarji under our supervision and has not been submitted elsewhere for a degree.

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ACKNOWLEDGEMENTS

I am happy to place on record my indebtedness to Dr. S.G. Dhande and Dr. B. Sahay who introduced me to the area of Computer-Aided Design and particularly to the subject of this thesis. Their excellent guidance and encouragement throughout the period, have led to the successful completion of this work.

I thank my colleagues K. Shantharam, H. Markale, P.L. Venkates, U. Muttur, S.K. Prasad and all others for their valuable co-operation and help during the preparation of this thesis.

I am thankful to Mr. D.P. Saini for careful and efficient typing of this thesis.

I wish to appreciate the excellent computer facilities provided by Computer Center, IIT, Kanpur for carrying out this work.

Shrinivas. Y. Umarji

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NOMENCLATURE

A	- Cross sectional area
A_s	- Equivalent shear area ($A_s = AK_s$ where K_s is the shear form factor)
C	- Concentrated moment
c	- Maximum distance from the neutral axis
c^*	- Moment intensity
d	- Diameter of circular cross section
E	- Modulus of elasticity
F_M	- Loading function for the bending moment
F_V	- Loading function for the shear force
F_y	- Loading function for the deflection
F_θ	- Loading function for the slope
FS	- Factor of safety
G	- Shear modulus of elasticity
I	- Moment of inertia
I_z	- Moment of inertia about Z axis
J	- Polar moment of inertia
K	- Stress concentration factor for normal stress
K_t	- Stress concentration factor for shear stress
K^*	- Elastic foundation modulus
K_1^*	- Rotary foundation modulus
K_1	- Translational spring constant
K_2	- Rotational spring constant
L	- Total length of the shaft
l	- Length along the shaft for a particular section or field matrix

M	- Bending moment at any section of the shaft
M_o	- Bending moment at left end of the shaft (x=o)
M_T	- Thermal moment
P	- Axial load
r	- Radius at any location
S	- Equivalent normal stress
$\{S\}$	- State vector at any position along the shaft
$\{S\}_o$	- State vector at the left end (x=o)
T	- Applied torque
t	- Time
$[U]$	- Global matrix
$[U_i]$	- Field matrix i
$[\bar{U}_i]$	- Point matrix i
V	- Shear force at any section of the shaft
V_o	- Shear force at left end of the shaft
w	- Distributed load
x	- Position axially along the shaft
x-y-z	- Coordinate system
y	- Deflection of shaft laterally
y_o	- Deflection at left end of shaft (x=o)
$\bar{y}$	- Centroid of the area above the y coordinate
z	- Distance from the neutral axis
$\frac{\Delta w}{\Delta l}$	- Gradient of distributed load or linearly varying load
θ	- Slope of the deflection curve or of the shaft

θ_o	-	Slope at the left end of shaft (x=o)
ρ	-	Mass per unit length
γ	-	Poisson's ratio
σ_{AX}	-	Normal stress for axial load
σ_B	-	Normal stress for bending
σ_x	-	Normal stress in x direction
σ_y	-	Normal stress in y direction
σ_e	-	Endurance limit
σ_{yp}	-	Yield stress
σ_{ut}	-	Ultimate strength
τ_{TORQ}	-	Shear stress due to torque
τ_{max}	-	Maximum shear stress
τ_{TR}	-	Transvers shear stress

ABSTRACT

In the present work, a methodology of interactive, design and analysis of multi-speed machine tool gear box shafts has been developed. Computer program developed here gives an appropriate design of gear box shafts selected from the different specified options. The program can also analyse general machine shafts, when geometric properties, material properties and general loadings are given as inputs. This analysis procedure provides complete information about the deflections, stresses, and the factor of safety with considerations of stress concentrations, failure due to yielding and fatigue. The program includes graphics display of results so as to help the engineer in the process of evaluating the design. The analysis procedure is based on transfer matrix method using the fundamental equations of motion for the bending of a beam. The proposed approach has been illustrated with the help of four case studies and the results obtained have been discussed.

CHAPTER - I

INTRODUCTION

1.1 Role of Gear Boxes:

The gear boxes of general purpose machine tools provide a wide range of cutting speeds and torques from a constant speed power input enabling proper cutting speeds or torques to be obtained at the spindle as required in the case of spindle drives and the desired feed rates in the case of feed drives. Optimum cutting speeds and feed rates enable the operator to obtain optimum rate of metal removal and minimum operation time.

With a constant speed power source there is a need for some method of varying the speed over a given range. Stepless mechanical and electrical drive can provide infinite speed variation. However, the torque/ speed characteristics of available stepless drives do not meet the requirements of spindle drives which demand an increased driving torque at lower output speeds in order to maintain a constant rate of metal removal. The stepless drives which do possess the required torque/ speed characteristics are limited by the speed range

over which these characteristics can be maintained. In order to provide a wide range of operating speeds together with adequate torque at lower spindle speeds, it is necessary to use gear boxes which enable the required spindle speed range to be covered in a number of discrete steps.

1.2 Design of Shafts in Gear Boxes:

The gear box consists of sliding gears, non-sliding gears, shafts, bearings and casing. Thus shafts are the integral part of a gear box and the design of these shafts is of prime importance.

Shafts are rotating members whose function is to transmit power and motion. Since shafts are so commonly used, it is very important to employ a good design procedure. Shaft design consists primarily of determining the correct shaft diameters to provide satisfactory strength and rigidity. Most shafts are generally subjected to fluctuating loads of combined bending and torsion with various degrees of stress concentration.

Since shafts are the most commonly designed machine elements, the assumptions and sequence of design steps are instrumental to the results of the final design. This design process must take into account applied forces,

permissible deflection limits and stress concentrations at critical sections and locations such as keyways, etc Most of the shafts can be treated as beams which enables usage of strength of material equations for deflection, slope, bending moments, and shear force along the shafts [1,2].

The gears must be suitably mounted on shafts of such size as properly to take care of maximum torque loads imposed by rated horsepower and speeds. Theoretically these shafts should be capable of withstanding as much load as can be transmitted by the gear teeth and must also be rigid enough to resist any bending loads that might possibly tend to affect proper engagement of the gear teeth. The shafts must also be provided with the necessary keyways, splines, etc. by means of which the motor and driven units can be connected.

Since horsepower is a function of the product of speed and torque, the high speed shaft is small and the low speed shaft is larger so as to give an increase in strength which is in proportion to the decrease in speed.

The shafts must be supported on bearings, usually of the ball or roller type, of such size, type and capacity as properly to care for the loads, both radial

and thrust, which would be setup under maximum operating conditions.

1.3 Objective of the Present Work:

With computers the design process of shafts or any machine element can be significantly improved. To be able to effectively utilize the computer, a correct and simple algorithm for shaft design must be developed in detail. By employing the computer, the results of minor adjustments or major changes in a design can be quickly assessed. This helps the engineer to test sev different designs without doing large amount of repetitive calculations manually. Also using interactive graphics, the engineer can examine the possible models on a computer terminal before executing the solution procedure.

Gear boxes with a large number of steps within a given range would be bulky and expensive. Hence these should be so designed that while fulfilling the functional requirements, the assembly should be economical to manufacture. The cost of the gear box is related to the number of shafts and bearings required and the total number and size of the gears. From different possible arrangements of the gears, the layout which gives compact size and lower cost while still fulfilling the technical requirement of the system should be chosen [3,4,5].

To reduce the cost of the gear box the optimal sizes of the shafts should be found out from different possible arrangements of the gears.

Once the number of gears, shafts and the number of teeth on each gear and also the input horsepower are known, one can search for the possible arrangements of the gears. In order to get minimum length and diameter of a shaft it should be kept in mind that the distance between two gears of the fixed block must be equal to two gear widths. Thus by giving possible different gear arrangements the loads on the intermediate nonsliding shafts can be calculated. By knowing the loadings, geometric properties such as diameters and bearing positions and material properties, the factor of safety and deflection along the shaft can be determined. This is essentially an analysis problem which determines how safe a shaft is for the proposed loading. After analysis of each possible gear arrangement, the best possible design of a shaft is determined.

The aim of this work is to develop the solution procedures and algorithms for the suggested problems. Since stress concentrations cannot be avoided, the algorithms incorporate stress concentration to provide an accurate solution technique. The major aim of this work

is to supply a reliable solution procedure. This permits an engineer to devote more time to the engineering aspect of shaft design and less time on repetitive calculations.

1.4 Scope and Limitations:

Most shafts are statically indeterminate systems which require both the equations of statics and deformation to be solved. These systems can be analysed by using finite difference, finite element, or transfer matrix methods. Using any of these methods enables modeling of shafts with complex loading and arbitrary cross-sections. But the present work deals with only circular cross-sections.

The transfer matrix method is chosen over the other two methods for several reasons [6]. The finite element method gives an accurate solution, but its solution procedure is technically more complex. Concerning programming capabilities, the finite element method requires greater amount of computer storage and is more difficult to program. When programming the finite difference method, more storage and more computer time is needed than for transfer matrix method. The transfer matrix method provides an efficient procedure for finding the deflection, slope, bending moment and shear

force at any location along the shaft [6,7]. A major advantage of this method is to model all possible types of loads, supports, and geometry, with a simple solution procedure that can be easily programmed. For example, a stepped shaft with different material properties and loadings along the shaft can be easily analysed. The possible types of loadings considered in this work are concentrated loads, distributed loads, torques, concentrated moments and axial forces. Supports can be located anywhere along the shaft with no theoretical limit on the number of supports. Rigid supports, translational and rotational springs also can be modeled. This permits rigid, pinned, flexible, and free boundary conditions to be considered.

The major types of stresses to which the shaft is subjected are shear stresses caused by torsional loading and shear forces, and normal stresses caused by axial loading and bending [1,8,10]. The shear forces and bending moments are computed by transfer matrix analysis, where as the torques and axial loads are known input data. The state of stress at each point along the shaft must be combined to form an equivalent stress by employing an appropriate failure theory. The types of failure theories commonly considered in design texts are the maximum normal stress theory, maximum shear stress theory

and the distorsion energy theory [8,9,10]. The maximum normal stress theory is generally recommended for brittle material and is not usually considered for shafting. So only the last two theories are considered here.

Since some shafts are subjected to fluctuating loads with nonzero means, the resulting stresses are also fluctuating with nonzero mean stress. However, most of the fatigue data available is from tests with zero mean stress i.e. completely reversed load. Therefore, it is important for the design equations to include a stress which has an equivalent zero mean cyclic stress. Several methods have been devised for estimating the equivalent zero mean stress when given general stress condition. These methods include the modified Goodman, Soderberg and Gerber relationships [9]. The Soderberg relationship is a conservative approximation and is one of the most commonly applied methods. Thus it has been incorporated in this work along with the two failure theories to calculate an equivalent or resultant uni-axial stress. Once the equivalent stress is found, the failure stress can be divided by the equivalent stress to produce the factor of safety at each desired location. From the factor of safety at the different stress concentrations and at other critical points along the shaft, the smallest factor of safety for a given shaft can be determined.

Shaft design requires determining the shaft diameters that ensure a safe design. Thus knowing all the loading, support conditions and the material properties the diameter of the shaft can be calculated. Thus in case of gear box the diameters and the lengths of the shaft for each possible different cases are found out and the smallest and best of all is chosen as the optimal shaft size.

CHAPTER - 2

TRANSFER MATRIX METHOD

2.1 Introduction:

For the design of rotating shafts, the shaft's strength and rigidity are important. Strength evaluation requires the calculation of stresses and stiffness considerations require the calculation of slope and deflection.

To calculate stress along the shaft the bending moment and shear force must be known. Stiffness of shaft is of interest because the slope of shaft can be a limiting case near the bearing support and excessive deflection might cause improper engagement of gears and interference with another machine parts.

By transfer matrix we can calculate simultaneously the deflection, slope,bending moment and shear force at any point along the shaft. In this work the differential equations developed assume small deflections and linear elastic material properties.

The transfer matrix approach for modelling of rotating shafts and beams as given by Pilkey and Chang[7]

is presented in this chapter. Thus the formulation deals with static stability analysis of beams by considering arbitrary loadings and several types of supports.

2.2 Governing Equations for Shaft Analysis:

Since a shaft can be assumed to be a rotating beam, the equations of motion for a beam as presented by Timoshenko [1] can be used to develop the transfer matrices. These general equations for the bending of beams as given by Pilkey and Chang [7] are,

$$\frac{\partial y}{\partial x} = -\theta + \frac{V}{GA_s} \tag{2.1}$$

$$\frac{\partial \theta}{\partial x} = \frac{M}{EI} + \frac{M_T}{EI} \tag{2.2}$$

$$\frac{\partial M}{\partial x} = V + (K_1^* - P)\,\theta - d\xi\, r_y^2\, \frac{\partial^2 \theta}{\partial t^2} - C\ (x,t) \quad \ldots \tag{2.3}$$

$$\frac{\partial V}{\partial x} = K^* y + \xi \frac{\partial^2 y}{\partial t^2} - w\ (x,t) \tag{2.4}$$

In this work presented here, these equations can be simplified by number of assumptions such as,

i) The shaft rests on a rigid foundation. Thus $K^* = K_1^* = 0$

ii) Since shafts are rotating, a radial thermal moment is unlikely to develop. Thus $M_T = 0$

iii) Since a moment intensity or distributed moment is not a common loading and which is not considered in this work. Thus $C\,(x,t) = 0$

iv) The axial loading effect on the motion of a beam is not significant for small deflections of shafts. Thus $P = 0$

v) When considering a Euler Bernoulli beam i.e. uniform ordinary beam the rotary inertia term is neglected. Thus $r_y^2 \frac{\partial^2 \theta}{\partial t^2} = 0$

vi) Since a static analysis is being considered, time dependance is also eleminated.

Considering these assumptions, the above equations 2.1 to 2.4 become

$$\frac{d\bar{y}}{dx} = - \theta + \frac{\bar{V}}{A_s} \tag{2.5}$$

$$\frac{d\theta}{dx} = \frac{\bar{M}}{EI} \tag{2.6}$$

$$\frac{dM}{dx} = V \tag{2.7}$$

$$\frac{dV}{dx} = w \tag{2.8}$$

Except A_s in above equations 2.5 to 2.8 all the terms are common to strength of materials. A_s is the equivalent shear area and can be calculated as

$$(A_s)_{SFF} = A\,K_s \tag{2.9}$$

where, A = Cross sectional area

K_s = Shear form factor (SFF)

This shear form factor K_s for some standard shapes can be found in reference [11,12]. For circular cross section it is

$$K_s = \frac{6\ (1 + \gamma)}{7 + 6\ \gamma} \tag{2.10}$$

where, γ = Poisson's ratio

After rearranging and integrating, Equations 2.5 to 2.8 become

$$y = y_o - \theta_o\ x - M_o\ \frac{x^2}{2EI} - V_o\ \left(\frac{x^3}{6EI} - \frac{x}{GA_s} \right) + F_y \tag{2.11}$$

$$\theta = \theta_o + M_o\ \frac{x}{EI} + V_o\ \frac{x^2}{2EI} + F_\theta \tag{2.12}$$

$$M = M_o + V_o\ x + F_M \tag{2.13}$$

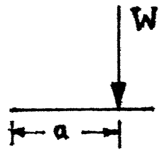
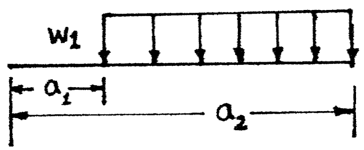
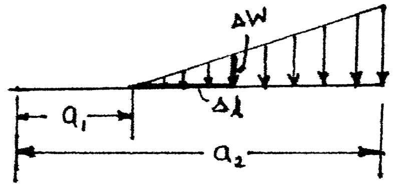
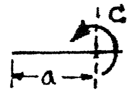
$$V = V_o + F_V \tag{2.14}$$

The loading functions, F_y , F_θ , F_M , F_V are given for general types of loading in Table 2.1. These loading functions are for the general solution of a shaft that is supported on the ends. If more than one loading occurs on the shaft, the loading function for each type of loading is added using superposition. The table uses singularity functions which state that for $n \geq 0$

$$\langle x - a \rangle^n = \begin{cases} 0 & \text{if} \quad x < a \\ (x-a)^n & \text{if} \quad x \geq a \end{cases} \tag{2.15}$$

TABLE 2.1

LOADING FUNCTIONS FOR GENERAL TYPES OF LOADING

				
$F_Y(x)$	$\frac{W\langle x-a\rangle^3}{3!\ EI}$	$\frac{w_1}{4!\ EI}(\langle x-a_1\rangle^4-\langle x-a_2\rangle^4)$	$\frac{1}{5!\ EI}\frac{\Delta W}{\Delta l}(\langle x-a_1\rangle^5-\langle x-a_2\rangle^5)$ $-\frac{\Delta W}{\Delta l}(a_2-a_1)\frac{\langle x-a_2\rangle^4}{4!\ EI}$	$\frac{C\langle x-a\rangle^2}{2\ EI}$
$F_\theta(x)$	$-\frac{W\langle x-a\rangle^2}{2\ EI}$	$-\frac{w_1}{3!\ EI}(\langle x-a_1\rangle^3-\langle x-a_2\rangle^3)$	$-\frac{1}{4!\ EI}\frac{\Delta W}{\Delta l}(\langle x-a_1\rangle^4-\langle x-a_2\rangle^4)$ $+\frac{\Delta W}{\Delta l}(a_2-a_1)\frac{\langle x-a_2\rangle^3}{3!\ EI}$	$-\frac{C\langle x-a\rangle}{EI}$
$F_M(x)$	$-W\langle x-a\rangle$	$-\frac{w_1}{2}(\langle x-a_1\rangle^2-\langle x-a_2\rangle^2)$	$-\frac{1}{3!}\frac{\Delta W}{\Delta l}(\langle x-a_1\rangle^3-\langle x-a_2\rangle^3)$ $+\frac{\Delta W}{\Delta l}(a_2-a_1)\frac{\langle x-a_2\rangle^2}{2}$	$-C\langle x-a\rangle^0$
$F_V(x)$	$-W\langle x-a\rangle^0$	$-w_1(\langle x-a_1\rangle-\langle x-a_2\rangle)$	$-\frac{1}{2}\frac{\Delta W}{\Delta l}(\langle x-a_1\rangle^2-\langle x-a_2\rangle^2)$ $+\frac{\Delta W}{\Delta l}(a_2-a_1)\langle x-a_2\rangle$	0

Equations 2.11 to 2.14 can be written in matrix notation as,

$$\begin{bmatrix} y \\ \theta \\ M \\ V \end{bmatrix} = \begin{bmatrix} 1 & -x & -x^2/2EI & -x^3/6EI + x/GA_s \\ 0 & 1 & x/EI & x^2/2EI \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_o \\ \theta_o \\ M_o \\ V_o \end{bmatrix} + \begin{bmatrix} F_y \\ F_\theta \\ F_M \\ F_V \end{bmatrix} \qquad (2.16)$$

or in extended form as

$$\begin{bmatrix} y \\ \theta \\ M \\ V \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -x & -x^2/2EI & -x^3/6EI + x/GA_s & F_y \\ 0 & 1 & x/EI & x^2/2EI & F_\theta \\ 0 & 0 & 1 & x & F_M \\ 0 & 0 & 0 & 1 & F_V \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} Y_o \\ \theta_o \\ M_o \\ V_o \\ 1 \end{bmatrix} \qquad (2.17)$$

Table 2.1 for loading functions F_y , F_θ , F_M and F_V for general uniform shafts as shown in table 2.1 . The column matrices are known as state vectors, since they contain the state variables y, θ , M and V. The state vector containing y_o , θ_o , M_o and V_o is located at the left end of the shaft. The square matrix is called the transfer matrix, since it 'transfers' the state vector at the left along the shaft. This transfer matrix can be represented in general form as,

$$[U_i] = \begin{bmatrix} U_{yy} & U_{y\theta} & U_{yM} & U_{yV} & F_y \\ U_{\theta y} & U_{\theta\theta} & U_{\theta M} & U_{\theta V} & F_\theta \\ U_{My} & U_{M\theta} & U_{MM} & U_{VM} & F_M \\ U_{Vy} & U_{V\theta} & U_{VM} & U_{VV} & F_V \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.18)$$

where each term in $[U_i]$ gives the influence of the indicated left end state variable on the indicated right end state variable. For example, $U_{y\theta}$ gives the influence of θ_o on y and is equal to - x in Equation (2.17).

When the shaft has a complicated loading or in-span supports, the simultaneous equations become complex and difficult to manipulate into a specific solution. However, the solution process for complicated shafts can be easily solved with the transfer matrix method.

2.3 Sign Convention:

Special attention must be given to the sign convention used in the development of the different types of transfer matrices, to give the solution. Using a right handed coordinate system the x axis is positive to the right, z axis is positive out of the paper and the y axis is positive downward as in Figure 2.1. All external forces are positive if applied in the direction

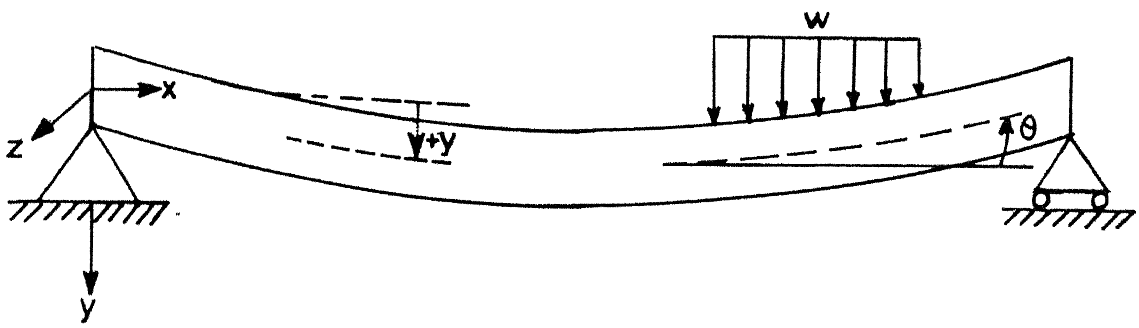


Fig.2.1 Positive displacement y and slope θ.

of a positive axis. For example, a concentrated force applied in the direction is positive downward. Likewise, internal forces and moments acting on a positive face (face on the right end of a free body diagram, see Figure (2.2))are positive when their vectors are in the direction of a positive coordinate. Internal forces and moments acting on a negative face are positive in the direction opposite the positive coordinate, as shown in Figure (2.2). Following this right hand rule, the deflection and slope are also positive if their vectors are in the positive coordinate direction as in Figure (2.1).

2.4 Types of Transfer Matrices:

There are two types of transfer matrices, field matrices and point matrices.

i) Field Matrices: These matrices are generated for finite lengths along the shaft with constant material properties and constant geometrical properties. The field matrix* for section of length l is given by $[U_i]$ as,

$$[U_i] = \begin{bmatrix} 1 & -l & -l^2/2EI & -l^3/6EI + l/GA_s & F_Y \\ 0 & 1 & l/EI & l^2/2EI & F_\theta \\ 0 & 0 & 1 & l & F_M \\ 0 & 0 & 0 & 1 & F_V \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots \quad (2.19)$$

* Derivation of field matrix is given in Appendix I.

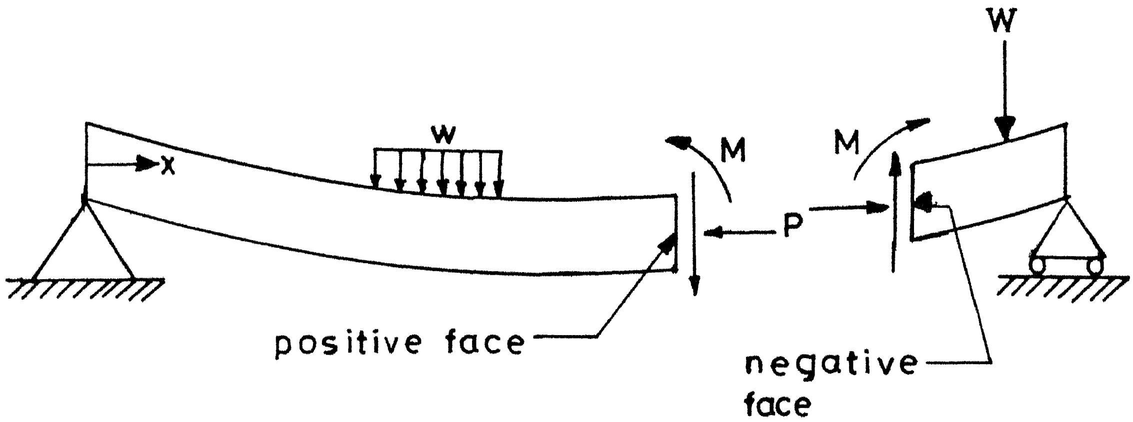


Fig. 2.2 Sign convention for positive bending.

The general form of a distributed load is given in Figure(2.3). The loading equations can be obtained from table 2.1 by letting $x = a_2$ and $l = a_2 - a_1$ and superposing the cases for a constant distributed load and a triangular distributed load. These equations for the loading functions reduce to

$$F_y = w \left(\frac{l^4}{24EI} - \frac{l^2}{2GA_s} \right) + \frac{\Delta w}{\Delta l} \left(\frac{l^5}{120EI} - \frac{l^3}{6GA_s} \right) \qquad (2.20)$$

$$F_\theta = - w \left(\frac{l^3}{6EI} \right) - \frac{\Delta w}{\Delta l} \left(\frac{l^4}{24EI} \right) \qquad (2.21)$$

$$F_M = - \frac{wl^2}{2} - \frac{\Delta w}{\Delta l} \left(\frac{l^3}{6} \right) \qquad (2.22)$$

$$F_V = - wl - \frac{\Delta w}{\Delta l} \left(\frac{l^2}{2} \right) \qquad (2.23)$$

where w = constant distributed load

$\frac{\Delta w}{\Delta l}$ = gradient of distributed load (Δw is the change in the loading and Δl is the change in length)

If section is not loaded, the loading functions are set equal to zero.

Field Matrices for Axial Load : These matrices are also generated for finite length along the shaft with constant material properties and constant geometric properties. Field matrix for any section of length l: for axial loading in given by $[U_i]$ as,

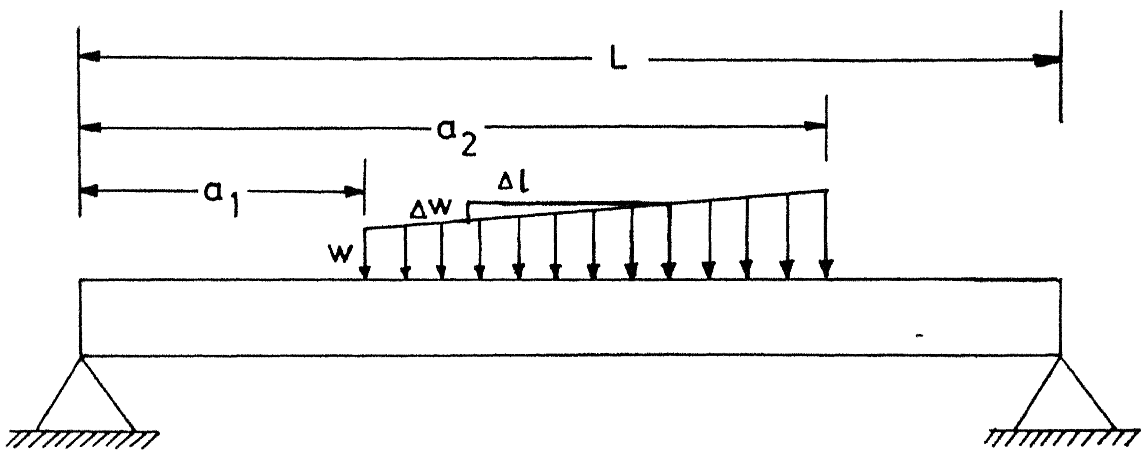


Fig. 2.3 Simply supported shaft with linearly varying distributed load.

$$[U_i] = \begin{bmatrix} 1 & -s & \frac{1-c}{d_1 EI} & \frac{1-s}{d_1 EI} + \frac{1}{GA_s} & F_y \\ 0 & c & \frac{s}{EI} & -\frac{1-c}{d_1 EI} & F_\theta \\ 0 & EIsd_1 & c & s & F_M \\ 0 & 0 & 0 & 1 & F_V \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots. \quad (2.24)$$

where a) For compressive axial force P

$$\alpha = \sqrt{\frac{P}{EI}}, \quad d_1 = -\alpha^2$$

$$s = \frac{\sin \alpha 1}{\alpha}, \quad c = \cos \alpha 1$$

b) For tensile axial force P

$$\alpha = \sqrt{\frac{-P}{EI}}, \quad d_1 = \alpha^2$$

$$s = \frac{\sinh \alpha 1}{\alpha}, \quad c = \cosh \alpha 1$$

$$F_V = w\left[\frac{1}{d_1 EI} \left(\frac{c-1}{d_1} - \frac{1^2}{2} \right) - \frac{1^2}{2GA_s} \right.$$

$$\left. + \frac{\Delta w}{\Delta 1} \left[\frac{1}{d_1 EI} \left(\frac{s-1}{d_1} - \frac{1^3}{6} \right) - \frac{1^3}{6GA_s} \right] \right.$$

$$F_\theta = w \left(\frac{1-s}{d_1 EI} \right) + \frac{\Delta w}{\Delta 1} \frac{1}{d_1 EI} \left(\frac{1^2}{2} + \frac{1-c}{d_1} \right)$$

$$F_M = w \left(\frac{1-c}{d_1} \right) + \frac{\Delta w}{\Delta 1} \left(\frac{1-s}{d_1} \right)$$

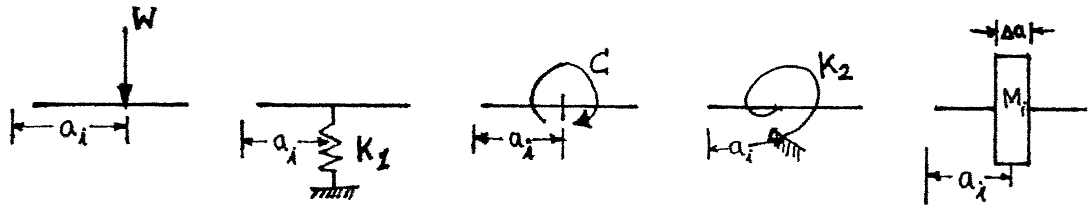
$$F_V = -w\,1 - \frac{\Delta w}{\Delta 1} \left(\frac{1^2}{2} \right)$$

P axial force is taken as the actual axial force in the segment as found from axial equilibrium requirements.

ii) Point Matrices: These matrices take into account the concentrates occurrences. Here a point occurrence is defined as a concentrated load or moment. The cases of concentrated occurrences considered in this work are point loads, concentrated moments, in-span supports, translational springs and rotational springs (and lumped masses). The point matrix $[\bar{U}_i]$, is presented for all of these cases except for in-span supports in Table 2.2. Noticing that for each type of point occurrence, there is only one entry to the identity matrix. For example the point matrix for a translational spring is formed by substituting the value of the spring constant K_1 for the (4,1) element in the (5 x 5) identity matrix given in Table 2.2 and where K_2 , C and W are zero. Since the term $I_p \omega^2$ in Table 2.2 is small compared to others, thus in the program developed, this term is not included. Usually the shafts are supported on bearings and the boundary condition is given as simply supported one. But if we know the mass of the bearing pedestal, stiffness coefficients and damping coefficients and whirl frequency then the boundary condition at the bearing can

TABLE 2.2

POINT MATRICES



$M_i = \Delta a \rho \quad I_{Pi} = \Delta a \rho\, r_p^2$

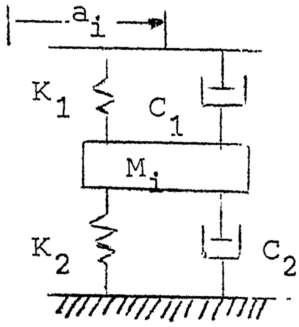
ω = frequency

ρ = mass/length

$r_p = \dfrac{d}{\sqrt{8}}$

$$[\bar{U}_i] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & K_2 + I_{Pi}\omega^2 & 1 & 0 & C \\ K_1 - M_i\omega^2 & 0 & 0 & 1 & -W \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

be given as free one but there will be a point matrix [7,14] which is as shown,



$$[\bar{U}_i] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ +Z_i & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.25)$$

where

$$Z_i = \frac{(K_1 + SC_1)\ (K_2 + SC_2 + S^2 M_i)}{K_1 + SC_1 + K_2 + SC_2 + S^2 M_i}$$

K_1 = Stiffness coefficient for bearing (force/length)

K_2 = Stiffness coefficient for bearing (force/length)

C_1 = Damping coefficient for bearing (force time/length)

C_2 = Damping coefficient for bearing (force time/length)

M_i = Mass of the bearing pedestal

S = $i\ \Omega$

Ω = Whirl frequency (radians/time)

2.5 Global Matrix:

After the transfer matrices are formed, the global matrix is formed by multiplying together the individual transfer matrices. The global matrix can be defined as,

$$\{S\}_{x=L} = [U_n]\ [U_{n-1}]\cdots[U_2]\ [\bar{U}_1]\ [U_1]\ \{S\}_{x=0} \quad (2.26)$$

which after matrix multiplication becomes

$$\{S\}_{x=L} = [U]\ \{S\}_{x=0} \quad (2.27)$$

where $[U]$ is a global matrix. Here, $\{S\}_{x=o}$ is the state vector which contains Y_o, θ_o, M_o and V_o. For the case in Equation (2.26), $[U_1]$ is the first field matrix at the left end of the shaft and $[\bar{U}_1]$ is a point matrix that occurs at the right end of field matrix $[U_1]$. The transfer matrices in Equation (2.26) are so arranged that first matrix $[U_n]$ is the right most matrix . Then matrix which is just to its left and so on till the $[U_1]$ left most matrix is reached. The matrices must be multiplied together from left to right, since matrix multiplication is not commutative. For example, when multiplying $[\bar{U}_1]$ $[U_1]$, the point matrix must be on the left side to obtain the correct answer. Once the global matrix has been found, the boundary conditions can be applied.

2.6 Boundary Conditions:

In this work three types of boundary conditions considered are fixed ends. Pinned ends and free ends. Pinned and fixed ends must have a rigid supports located at the specified end. A fixed end is a condition for which the deflection and slope are equal to zero. In case of pinned end the deflection equal to zero, but since a pinned end is free to rotate, the bending moment is equal to zero at that location. In case of free end, both the bending moment and shear force are zero. If a shaft has a spring support or point load at the

end of an overhung section, this occurrence is modelled as a point matrix on a free end.

The purpose of applying the boundary conditions is to solve the governing equations and to obtain the state vector at the left end. Two state variables are always known at the left end from the boundary conditions, and the other two state variables are calculated from the boundary conditions at the right end. Two equations with the unknown state variables are formed from the global matrix and the four boundary conditions. A simple procedure to calculate these equations has been developed by Pilkey [7], and the same is used in this work. In this procedure, the columns of the global matrix are cancelled for the state variables that are zero at the left end. For example, if the deflection is zero at the left end, the first column of the global matrix is cancelled. Rows are cancelled where the state variables at the right end are unknown. After this two equations remain which contains two unknown state variables at the left end can be solved and values of the unknowns can be found.*

To illustrate this procedure, a shaft is considered with a pinned end at the right end and with a fixed end at the left end, as shown in Figure (2.4).

* A problem is solved in Appendix II, to illustrate the procedure.

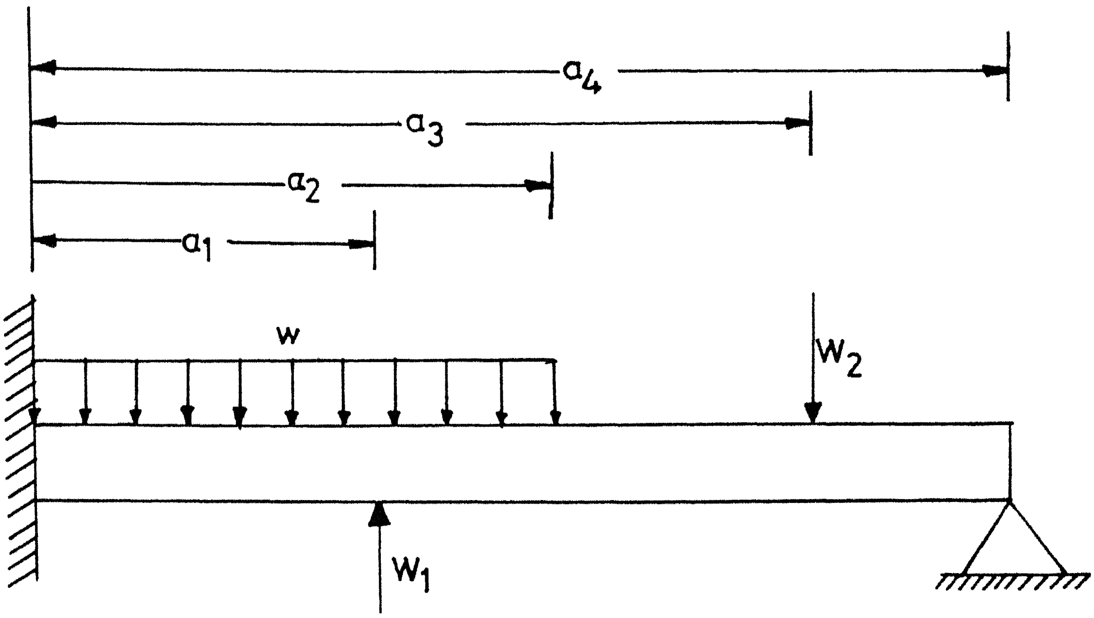


Fig. 2.4 Loaded shaft used for application of boundary conditions.

The transfer matrix notations for this shaft becomes,

$$\{S\}_{x=a_4} = [U_4]\ [\bar{U}_3]\ [U_3]\ [U_2]\ [\bar{U}_1]\ [U_1]\ \{S\}_{x=o} \quad (2.28)$$

The general global matrix obtained after matrices multiplied together in Equation (2.28) is as,

$$\begin{bmatrix} y = o \\ \theta \\ M = 0 \\ V \\ 1 \end{bmatrix}_{x=L} = \begin{bmatrix} U_{yy} & U_{M\theta} & U_{yM} & U_{yV} & F_y \\ U_{\theta y} & U_{\theta\theta} & U_{\theta M} & U_{\theta V} & F_{\theta} \\ U_{My} & U_{MO} & U_{MM} & U_{MV} & F_M \\ U_{Vy} & U_{V\theta} & U_{VM} & U_{VV} & F_y \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y_o = 0 \\ \theta_o = 0 \\ M_o \\ V_o \\ 1 \end{bmatrix}_{x=o} \quad \ldots \quad (2.29)$$

Since the deflection and slope are zero at the left end, columns 1 and 2 of the global matrix are eliminated. Similarly, with $\theta_{x=L}$ and $V_{x=L}$ as unknowns at the right end, where L is the length of the shaft, rows 2 and 4 of the global matrix are eliminated. This leaves the following two equations

$$0 = U_{yM}\ M_o + U_{yV}\ V_o + F_y \quad (2.30)$$

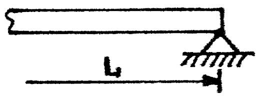
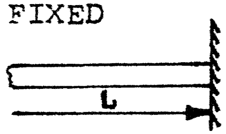
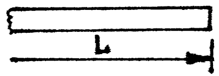
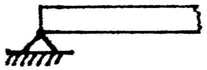
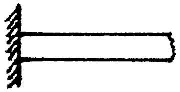
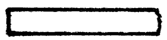
and

$$0 = U_{MM}\ M_o + U_{MV}\ V_o + F_M \quad (2.31)$$

Solving these two equations for M_o and V_o, yields the unknowns in the state vector at the left end. The initial parameters i.e. the unknowns state vector parameters can be calculated directly from Table 2.3 with

TABLE 2.3

INITIAL PARAMETERS FOR BEAMS WITH NO IN-SPAN SUPPORTS

LEFT END \ RIGHT END	1. PINNED 	2. FIXED 	3. FREE 
1. PINNED  $Y_o=0$; $M_o=0$	$\theta_o=(F_M U_{yV}-F_V U_{MV})/\nabla$ $V_o=(F_V U_{M\theta} F_M U_{yV})/\nabla$ $\nabla=U_{y\theta}U_{MV}-U_{M\theta}U_{yV}$	$\theta_o=(F_\theta U_{yV}-F_y U_{\theta V})/\nabla$ $V_o=(F_y U_{\theta\theta}-F_\theta U_{y\theta})/\nabla$ $\nabla=U_{y\theta}U_{\theta y}-U_{\theta\theta}U_{yV}$	$\theta_o=(F_V U_{MV}-F_M U_{VV})/\nabla$ $V_o=(F_M U_{V\theta}-F_V U_{M\theta})/\nabla$ $\nabla=U_{M\theta}U_{VV}-U_{V\theta}U_{MV}$
2. FIXED  $Y_o=0$; $\theta_o=0$	$M_o=(F_M U_{yV}-F_y U_{MV})/\nabla$ $V_o=(F_V U_{MM}-F_M U_{yM})/\nabla$ $\nabla=U_{yM}U_{MV}-U_{MM}U_{yV}$	$M_o=(F_\theta U_{yV}-F_y U_{\theta y})/\nabla$ $V_o=(F_y U_{\theta M}-F_\theta U_{yM})/\nabla$ $\nabla=U_{yM}U_{\theta y}-U_{\theta M}U_{yV}$	$M_o=(F_V U_{MV}-F_M U_{VV})/\nabla$ $V_o=(F_M U_{VM}-F_V U_{MM})/\nabla$ $\nabla=U_{MM}U_{VV}-U_{VM}U_{MV}$
3. FREE  $M_o=0$; $V_o=0$	$Y_o=(F_M U_{y\theta}-F_y U_{M\theta})/\nabla$ $\theta_o=(F_y U_{My}-F_M U_{yy})/\nabla$ $\nabla=U_{yy}U_{M\theta}-U_{My}U_{y\theta}$	$Y_o=(F_\theta U_{y\theta}-F_y U_{\theta\theta})/\nabla$ $\theta_o=(F_y U_{\theta y}-F_\theta U_{yy})/\nabla$ $\nabla=U_{yy}U_{\theta\theta}-U_{\theta y}U_{y\theta}$	$Y_o=(F_V U_{M\theta}-F_M U_{V\theta})/\nabla$ $\theta_o=(F_M U_{Vy}-F_V U_{My})/\nabla$ $\nabla=U_{My}U_{V\theta}-U_{M\theta}U_{Vy}$

the help of boundary conditions for shafts without in-span supports.

2.7 Calculation of Deflection, Slope, Bending Moment and Shear Force Along the Shaft:

Once the state vector, $\{S\}_{x=o}$ has been found, the deflection, slope, bending moment and shear force can be calculated anywhere along the shaft. Between sections, results are computed by appropriately adjusting the length variable in the field matrix for the point of interest. To calculate the deflection, slope, bending moment and shear force at a point of interest, the transfer matrices from the left end of the shaft to the point of interest are multiplied together with the state vector at the left end. For example in Figure 2.4., the state vector at $x = a_2$ is equal to

$$\{S\}_{x=a_2} = [U_2]\,[\bar{U}_1]\,[U_1]\,\{S\}_{x=o} \qquad (2.32)$$

While programming this procedure, it is not necessary to store all of the matrices. When initially calculating the transfer matrices, these matrices are multiplied together as they are developed. Significant parameters, such as moment of inertia, and modulus of elasticity, are stored for the later regeneration of the field matrices. No extra parameters are stored for the point matrices, since all of the required information is input data.

Once the global matrix is developed the state vector at left end of the shaft is calculated according to procedure developed using boundary conditions. When making a second pass to compute the forces and deflections along the shaft, the field matrices are regenerated to the appropriate length to determine the deflection, slope, bending moment, and shear force at the desired locations. However, when encountering a new field matrix, the state vector at the beginning of the new field matrix is calculated and is stored. This new state vector is used to calculate the deflection, slope, bending moment and shear force from that point on the shaft to the next new field matrix [14]. For example in Figure 2.4, the state vector at $x = a_2$ and $x = a_3$. In equation form this can be written as

$$\{S\}_{x = a_3} = [U_p] \quad \{S\}_{x=a_2} \tag{2.33}$$

where $[U_p]$ is the third field matrix having an appropriate adjusted length. This process saves remultiplying the matrices for the second sweep across the shaft.

2.8 In-span Supports:

A rigid support which does not occur at the end of the shaft is called an in-span indeterminate condition. The major differences in solving a shaft with

is used to calculate the deflection, slope, bending moment and shear force between $x = a_2$

an in-span support are the steps in the development of the global matrix and the application of the boundary conditions.

The solution procedure for in-span rigid supports is somewhat parallel to the case without in-span rigid supports. First, all the transfer matrices are calculated except for the in-span point matrices. Then, the subglobal matrices are determined from the calculated transfer matrices. Each subglobal matrix contains all of the transfer matrices between the left end and the first in-span support, between each in-span support, and between the last in-span support and the right end. Thus the number of subglobal matrices is equal to one more than the number of in-span supports.

For example, if two in-span supports are present along the shaft, then there are three subglobal matrices. The first subglobal matrix contains the transfer matrices from the left end to the first in-span support at $x = L_1$, as shown in Figure (2.5). The transfer matrices are multiplied together using the same method as for the global matrix. The second subglobal matrix contains the matrices from just to left of the first rigid in-span support to the second in-span support at $x = L_2$. The final subglobal matrix is determined in the same manner

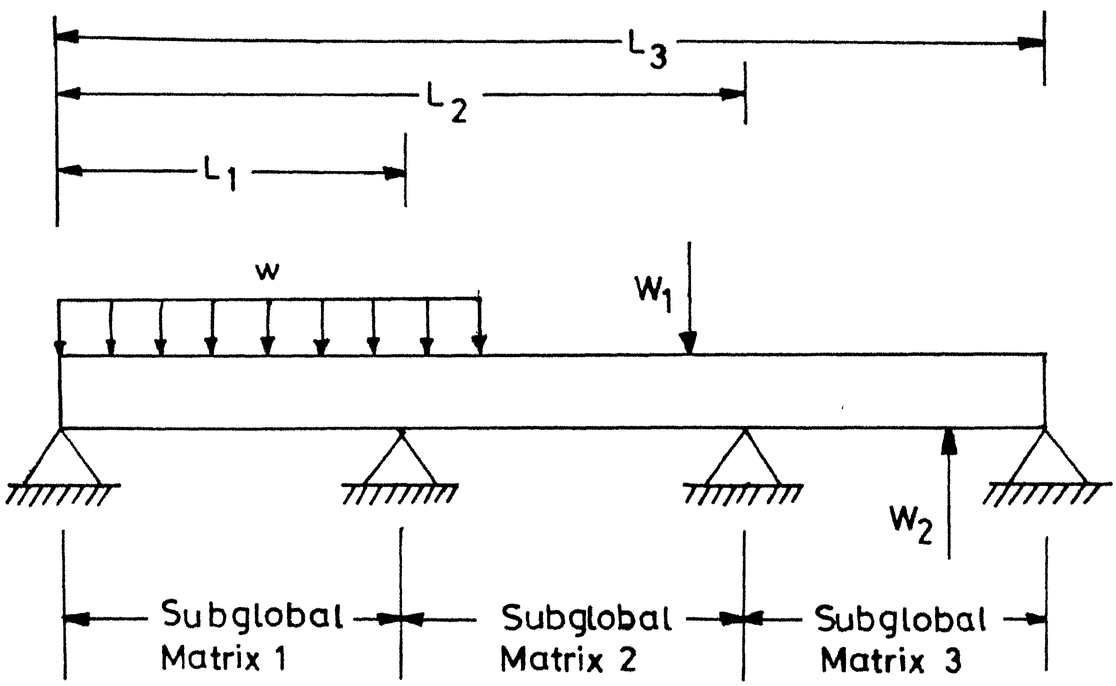


Fig. 2.5 Shaft used for determination of subglobal matrices.

from $x = L_2$ to the right end of the shaft. After the subglobal matrices have been calculated, the point matrices for the in-span supports can be determined. Solving for the point matrix at the in-span support is essentially equivalent to evaluating the reaction force at that location. The general form of the point matrix for the in-span supports is shown in table 2.4. To solve for this point matrix, the procedure uses a boundary condition to the right of the in-span support. If another in-span support occurs to the right of a given in-span support, the known condition is the deflection, and the subscript S_m in Table 2.4 is equal to y. However, if no in-span support occurs to the right of an in-span support, then subscript S_m in Table 2.4 is equal to one of the boundary conditions at the right end. The terms in the point matrix for in-span support are taken from the subglobal matrix to the right of the in-span support. The global matrix is equal to

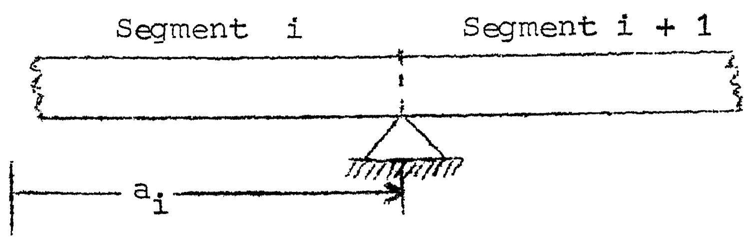
$$[U_6][\bar{U}_5][U_5][\bar{U}_4][U_4][\bar{U}_3][U_3][U_2][\bar{U}_1][U_1]$$

where $[\bar{U}_1]$ and $[\bar{U}_4]$ are point matrices for in-span supports. The subglobal matrices are

Subglobal matrix	Transfer matrices
1	$[U_1]$
2	$[U_4]\ [\bar{U}_3]\ [U_3]\ [U_2]$
3	$[U_6]\ [\bar{U}_5]\ [U_5]$

TABLE 2.4

Point matrix for in-span rigid support



U_{kj} k\j	y	θ	M	V	F
y	1	0	0	0	0
θ	0	1	0	0	0
M	0	0	1	0	0
V	$-\dfrac{U_{smY}}{U_{smV}}$	$-\dfrac{U_{sm\theta}}{U_{smV}}$	$-\dfrac{U_{smM}}{U_{smV}}$	0	$-\dfrac{U_{smF}}{U_{smV}}$
	0	0	0	0	1

Definition for rigid support:

$s_m = y$ if there is a rigid in-span support to the right of $x = a_i$. If there no in-span support to the right of a_i, then s_m is one of the state variables (y, θ, M, or V) that is zero at the right end of the beam i.e. to say that s_m is one of the boundary conditions at right end of the beam. U_{Kj} are the subglobal matrix elements for the segment between $x = a_i$ and the next in-span support or the right end.

Once the in-span point matrices are found, the global matrix is calculated by matrix multiplication in the order as discussed in Section 2.5. Since one of the boundary conditions at the right end is used for calculating the point matrix at the last in-span indeterminate condition, the method of applying boundary condition in this case is somewhat different from without in-span support. The four boundary conditions needed to solve the system equations are two at the left end, one at right end unused boundary condition and one at the first in-span support. The two simultaneous equations for calculating the two unknown parameters at the left end are formed in similar way that of in Section 2.6. One equation is developed from the global matrix and the unused boundary condition at the right end. The other equation can be formed from the first subglobal matrix and the known deflection at the first in-span support. After the state vector at left end is found, determining of the deflection, slope, bending moment and shear force is done using same procedure discussed in Section 2.7.

2.9 Transfer Matrices for Torsion:

Torsion is the result of one or more forces twisting a bar about a longitudinal axis. The net result of twisting loads, acting on a member is a couple

which has the dimensions of force and distance. The force unit is usually stated first to distinguish a torque from a bending moment which has the same dimensions.

If a transverse force is applied at such a point in the cross-section that twisting results, two equal and opposit forces act at the center of twist, with the result that a bending force and a couple act on the section. The center of twist is located at the shear center when a transverse force causes twisting.

For analysis of gear box shafts the torsion plays an important role.

The general form of transfer matrix for torsion is as

$$[U_i] = \begin{bmatrix} U_{\phi\phi} & U_{\phi T} & F_{\phi} \\ U_{T\phi} & U_{TT} & F_T \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.34)$$

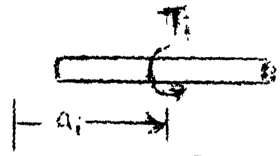
The field matrix for torsion is as

$$[U_i] = \begin{bmatrix} 1 & \frac{1}{GJ} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.35)$$

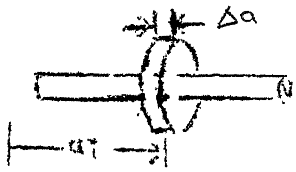
F_{ϕ} and F_T the loading functions are zero since only concentrated occurrence of torque is assumed, which is taken care in point matrix.

The point matrix for torsion is as

$$[\bar{U}_i] = \begin{bmatrix} 1 & 0 & 0 \\ -I_p\omega^2 & 1 & -T_i \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.36)$$



Concentrated applied torque



Disk occurrence $I_p = \rho r_p^2 \Delta a$

The boundary conditions in this case are only two, one is fixed and free. Since pinned end is free to rotate the boundary condition in case of pinned end is given as free end for torsional calculation. At fixed end angle of twist is zero and at free end torque is zero. The global matrix is found in similar way using the procedure discussed in Sections 2.5 to 2.7. And by applying boundary conditions the state vector $\{S\}_{x=o}$ is found out.

$$\{S\}_{x=o} = \begin{bmatrix} \phi_o \\ T_o \\ 1 \end{bmatrix} \qquad (2.37)$$

Then the angle at twist and torque anywhere along the shaft are found out using same procedure developed in Section s 2.5 to 2.7.

CHAPTER - 3

STRESS ANALYSIS

3.1 Introduction:

The intention of this chapter is to introduce the guide lines for finding the stresses on a rotating shaft and thus the least factor of safety. In considering the three dimensional loading on a shaft, the forms of stresses generated are shear stresses and normal stresses. The loadings considered on the shaft in this present work are, radial loading in two planes, axial loading and torsion. The shear stresses are developed by the torsional loading and radial shear forces (transverse shear). Likewise, the normal stresses are generated by the axial loading and the bending moments.

A mechanical element (say shaft) may be stressed in tension, compression or shear, either singly or in combination, by loads which are either static, fluctuating or composed of static and variable components. Depending upon the nature and magnitude of the loads, failure may occur by yielding, cracking or rupture. Statically loaded structures will normally perform satisfactorily

until the yield point of the material is reached and in some cases, until rupture occurs. Predicting failure for a shaft or any other mechanical element with uniaxial loading is relatively easy. However, for the possible loading on shafts, failure prediction is sufficiently more difficult because of a multiaxial state of stress. Several failure theories have been advanced for predicting failure under biaxial and triaxial stress conditions from uniaxial material properties [11]. Some assumptions are made while calculating stresses, bending moments and shear forces.

1) The beam (shaft) is of homogeneous material which has the same modulus of elasticity in tension and compression.

2) The shaft is straight.

3) The cross section is uniform.

4) All loads and reactions are perpendicular to the axis of the beam and lie in the longitudinal plane of symmetry.

5) The maximum stress does not exceed the preportional limit.

6) The beam is loaded only by equal and opposite twisting couples, which are applied at its ends in planes normal to its axis.

3.2 Shear Stress:

Since the shear stresses are developed by the torsional loading and radial shear forces (transverse shear), the shear stresses developed due to torsional loading are analysed as follows:

When the torque is applied to the shaft it twists, each section rotating about the longitudinal axis. Plane sections remain plane and radii remain straight. Thus at any point the shear stress τ_{TORQ} is developed on the plane of the section, the magnitude of this stress is proportional to the distance from the center of the section and its direction is perpendicular to the radius drawn through the point. Along with this shear stress there is an equal longitudinal shear stress on a radial plane, and equal tensile and compressive stresses at 45°. In addition to these stresses, the elongation of the outer fibers due to twisting causes longitudinal tension over the outer half of the section area and longitudinal compression over the inner half, the total tension and total compression being equal. The maximum longitudinal tensile stress occurs at the surface, the equal maximum longitudinal compressive stress occurs at the center. For the conditions here assumed, these longitudinal stresses are so small as

to be negligible in practically all cases [9].

Thus shear stress τ_{TORQ} in a shaft due to torque T varies uniformly from zero at the center to maximum at the outside. The maximum shear stress takes place at the outer surface i.e. when $r_o = d/2$ and is given as

$$\tau_{max} = \frac{T\, r_o}{J} \tag{3.1}$$

Since most of the shafts have solid, circular cross-sections, the polar moment of inertia is equal to,

$$J = \frac{\pi\, d^4}{32} \tag{3.2}$$

Thus maximum torsional shear stress for solid, circular cross-sections becomes

$$\tau_{max} = \frac{16T}{\pi d^3} \tag{3.3}$$

A shear stress is also created by the shear force or transverse loading. First, a shaft is assumed to be modelled as lengthwise elements. When the shaft is loaded, the elements within the shaft try to slide on each other, but since these elements are rigidly attached a shear stress is developed.

For any cross-section, the maximum transverse shear stress occurs at the neutral axis, for circular cross-section the maximum transverse shear stress is given as,

$$\tau_{max} = \frac{4}{3} \frac{V}{A} \tag{3.4}$$

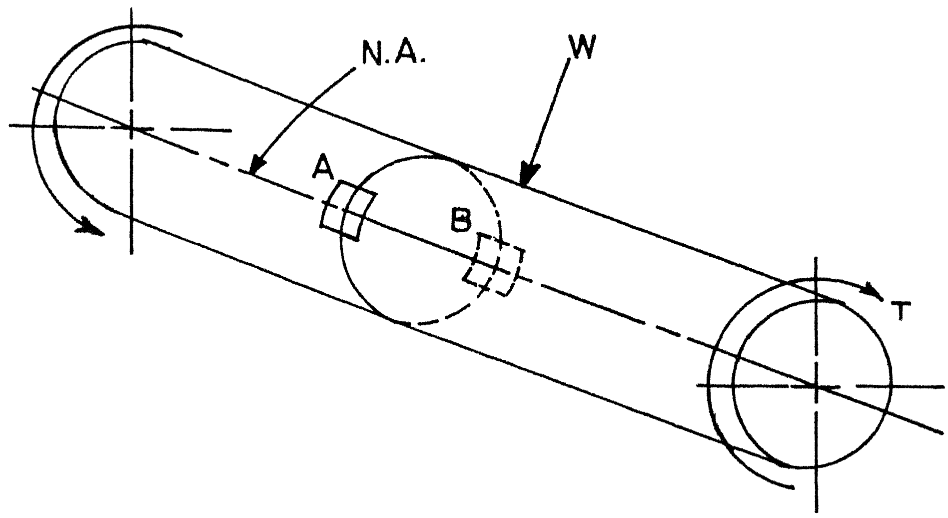
where, V = Shear force

A = Cross-sectional area.

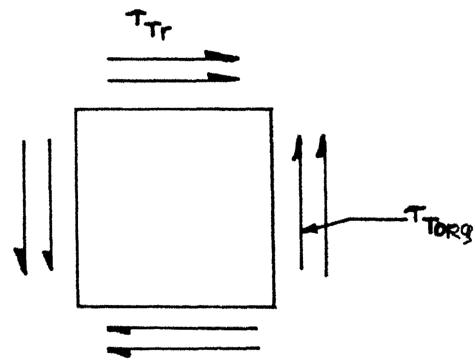
Since the transverse shear and torsional shear stress are in the same plane, they can be combined into one stress. Assuming constant torque for a circular cross-section, the maximum torsional shear stress is at any location on the circumference. However, the transverse shear stress ranges from zero at the outer fibers to a maximum at the neutral axis. The location of the neutral axis depends on the direction of the shear force, as shown in Fig. (3.1). Since the transverse shear is a maximum at the neutral axis, the limiting cases for shear stress occur at the outer fibers, as indicated in Fig. (3.1). Depending on the directions of the loads, the shear forces add on one side and subtract on the other. Since the shaft rotates, each point on the outer surface experiences a fluctuating nonzero mean shear stress.

3.3 Normal Stress:

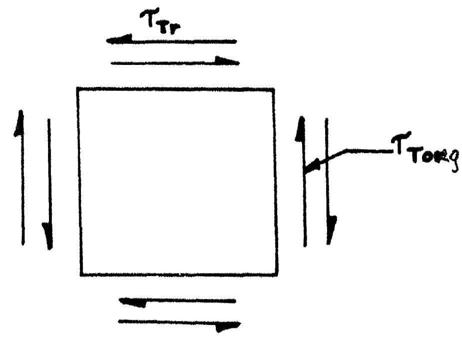
The normal stress is sometimes defined as the force per unit area. For a shaft, a normal stress is produced by an axial force and a bending moment. The normal stress due to an axial force, P, for a solid,



a) Shaft with torque, T, and applied load, W.



b) Element A, shear stresses add.



c) Element B, shear stresses subtract.

Fig. 3.1 Location of maximum shear stress.

circular cross-section is,

$$\sigma_{AX} = \frac{4\ P}{\pi d^2} \qquad (3.5)$$

where,

σ_{AX} = normal stress for axial load

d = diameter of cross-section

When normal stress due to bending is considered one side of the shaft is in tension, while the other side is in compression. The surface where stress is zero, is known as neutral axis. The maximum normal stress for bending for solid circular cross-section is

$$\sigma_{max} = \frac{32\ M}{\pi\ d^3} \qquad (3.6)$$

Combining the two normal stresses is similar to the case with two shear stresses. If the axial force for the shaft is assumed not to vary, the forces produce a constant normal stress across the cross-section. At any cross-section, the normal stress developed by the bending moment is zero at the neutral axis to a maximum at the outer edges. For each cycle that the shaft rotates, any one point on the outer surface experiences a maximum compressive stress and a maximum tensile stress from the bending moment. Therefore, the shaft has a non-zero mean fluctuating load. If the shaft is not axially loaded, the shaft develops a zero mean

fluctuating load [12]. The limiting cases are the two locations presented in Fig. (3.2).

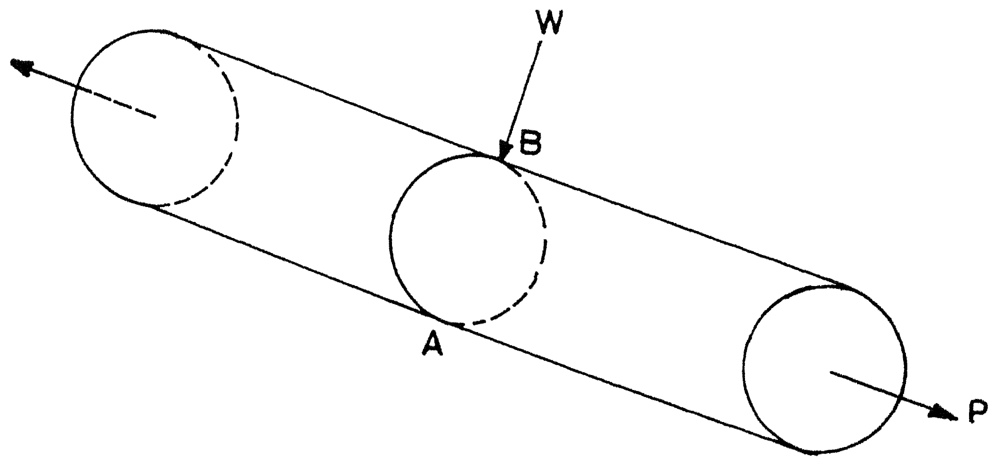
3.4 Stress Concentrations:

Failure generally is initiated at locations of geometrical discontinuities, which are known as stress concentrations. These geometrical discontinuities, such as grooves, keyways, fillets, and abrupt changes, cause irregularities in the stress patterns. A stress concentration factor is defined as

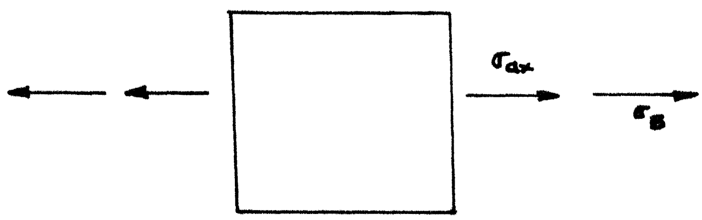
$$K = \frac{\text{Actual stress at cross-section}}{\text{Nominal stress at cross-section}} \qquad \ldots \quad (3.7)$$

Many methods have been used to find these stress concentration factors for different types of geometry and loading. These values of stress concentration factors can be found from various references. For example, Fig. (3.3) to (3.5) are taken from Peterson [13], and mathematical equations are given by Roark [9].

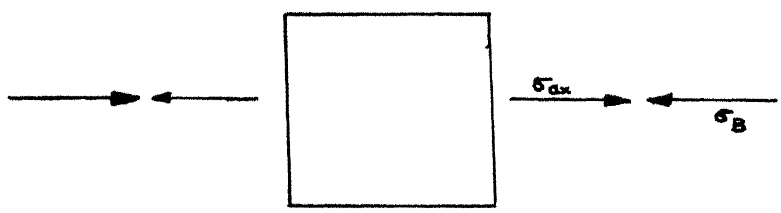
Considerations must be given to stress concentrations, since it is almost impossible to design a shaft without them. Minimizing the effects of stress concentrations becomes a necessity for an acceptable design. Some ways to decrease stress concentration effects for shafts are to use large fillet radii, to add grooves and to place stress concentrations in low stressed areas.



a) Shaft with axial load, P, and applied load, W.



b) Element A, normal stresses add.



c) Element B, normal stresses subtract.

Fig. 3.2 Location of maximum normal stress.

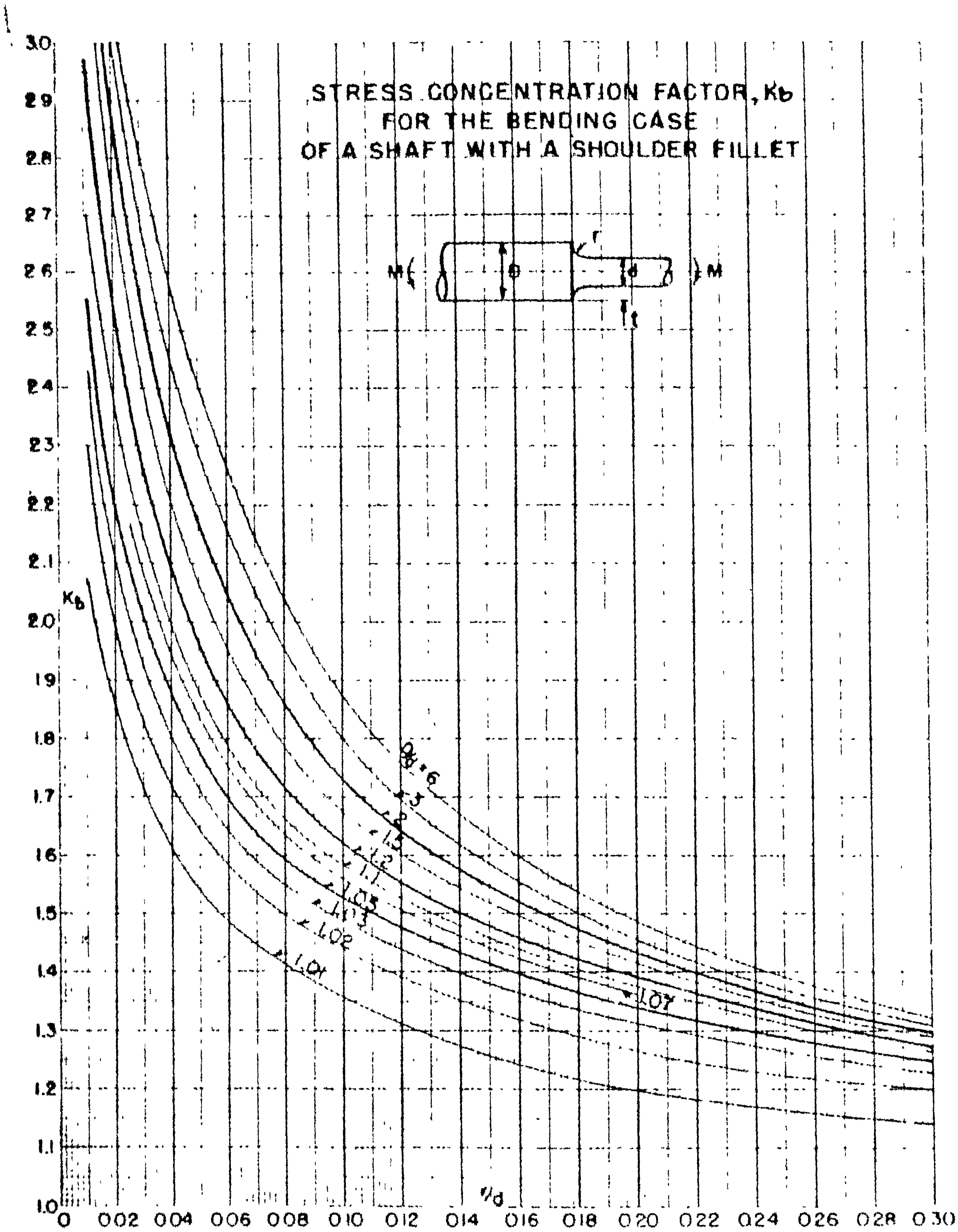


Fig. 3.3 Stress concentration factor K_b for bending of shaft with shoulder fillet. Ref. [13].

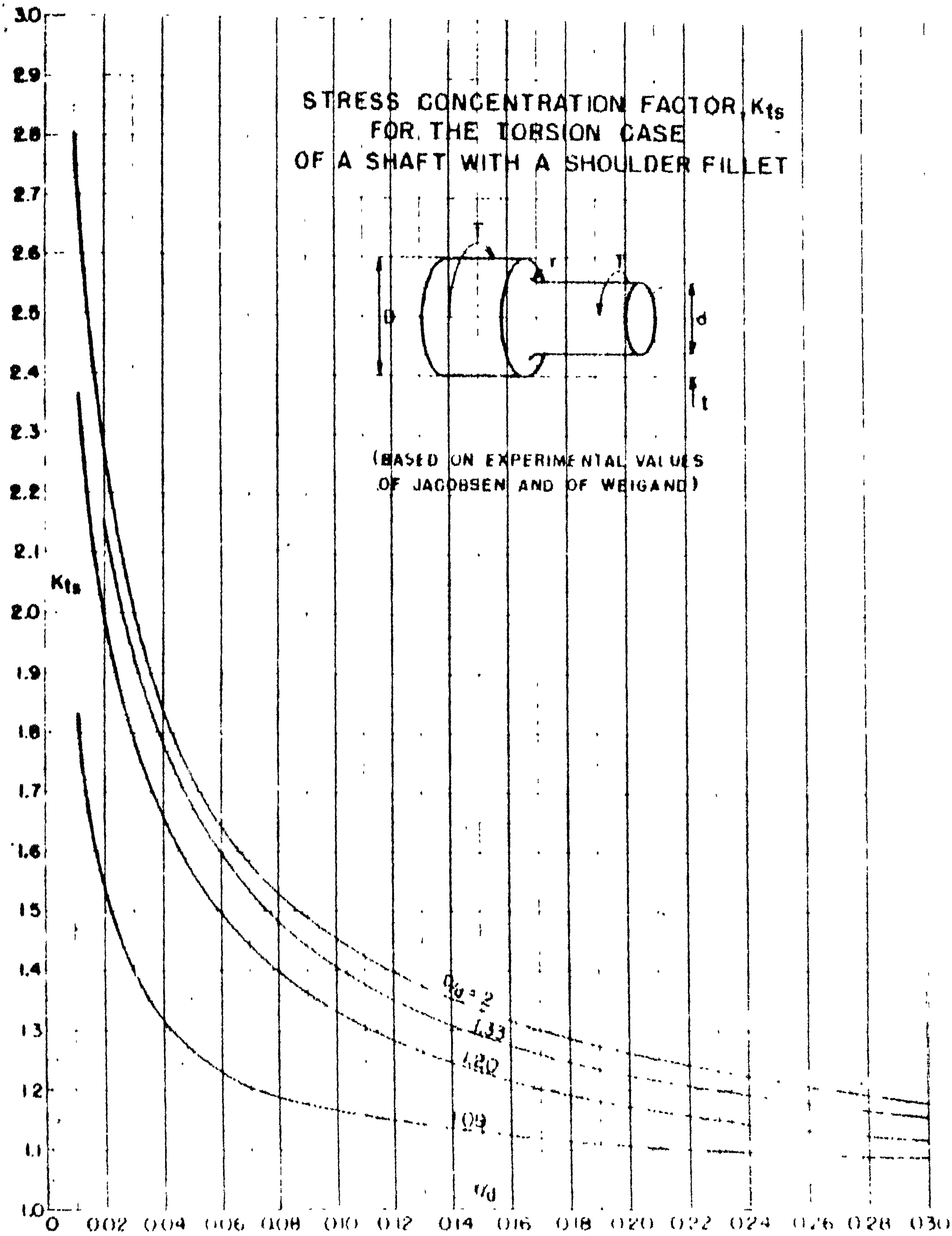


Fig. 3.4 Stress concentration factor K_{ts} for torsion of shaft with shoulder fillet. Ref.[13].

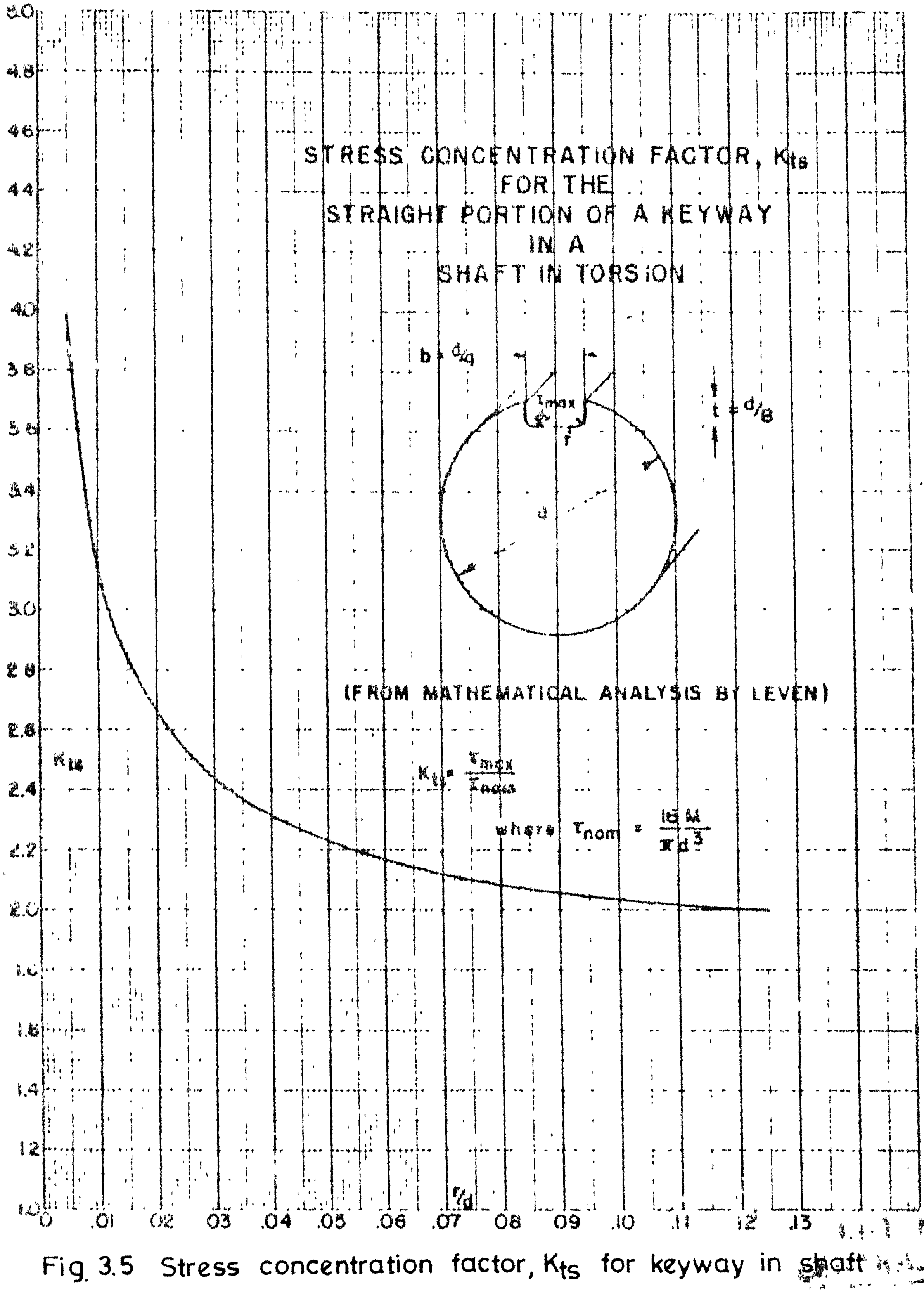


Fig. 3.5 Stress concentration factor, K_{ts} for keyway in shaft in torsion. Ref. [13].

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Effects of the stress concentrations can be quickly evaluated with a computer program. Also, a major advantage of a program is its ability to show locations of low stress for placement of stress concentrations.

3.5 Failure Theories:

Before introducing the failure theories, an expression must be developed to combine a steady and alternating stress to an equivalent steady stress. There have been many equations developed for test data to relate non-zero mean alternating stress to an equivalent zero mean alternating stress. The most conservative equation to perform this transformation is a Soderberg equation [8,10].

For normal stress, the Soderberg equation is

$$\sigma = \sigma_{av} + \frac{K\,\sigma_{yP}}{\sigma_e}\,\sigma_r \qquad (3.8)$$

where,

σ_{av} = average normal stress

σ_r = 1/2 normal stress range

σ_{yP} = materials yield stress

σ_e = materials endurance limit for zero mean stress loading

K = stress concentration factor

For shear stresses, the Soderberg equation becomes

$$\tau = \tau_{av} + \frac{K_t \sigma_{yP}}{\sigma_e} \tau_r \qquad (3.9)$$

where τ_{av}= average shear stress

τ_r= 1/2 shear stress range

K_t= stress concentration factor for shear

Equations (3.8) and (3.9) take into account stress concentrations. However, the stress concentration factors are usually applied only to the alternating stresses. The stress concentrations in the two equations are different. For example in equation (3.8), K is for bending, whereas K_t in equation (3.9) is for a torsional load. Since a rotating shaft is under consideration, the average normal stress for equation (3.8) is equal to the stress due to the axial loading. The amplitude of the alternating normal stress is produced by the bending moment and by any variations in the axial loading. However, the bending stress does not have to be completely reversed, but for the worst possible design case, the bending stress would be totally reversible. For equation (3.9), the average shear stress is due to the constant torque. The amplitude of the alternating shear stress is equal to the transverse shear or to any fluctuation in the torsional shear stress. Again, for the worst possible case, the transverse shear is presumed to be completely reversed.

Since most shafts are made of ductile materials, the two failure theories most accurately representing them are the maximum shear stress and distorsion energy failure theories. The distorsion energy failure theory fits experimental results for ductile materials extremely well [11] and [9].

For stresses in two dimensions, the theory gives

$$S^2 = \sigma_x^2 - \sigma_x \cdot \sigma_y + \sigma_y^2 + 3\,\tau_{xy}^2 \qquad (3.10)$$

where S = equivalent normal stress

σ_x = normal stress in x-direction

σ_y = normal stress in y-direction

τ_{xy} = shear stress in x,y plane.

For the case of rotating shaft, equation (3.10) reduces to

$$S^2 = \sigma^2 + 3\tau^2 \qquad (3.11)$$

After substituting equations (3.8) and (3.9) into the equation (3.11) and taking the square root, equation (3.11) becomes,

$$S = \sqrt{\left(\sigma_{av} + \frac{K\,\sigma_{yP}}{\sigma_e}\,\sigma_r\right)^2 + 3\left(\tau_{av} + K_t\,\frac{\sigma_{yP}}{\sigma_e}\,\tau_r\right)^2} \qquad \ldots \quad (3.12)$$

The working stress for uniaxial loading can be defined as

$$\sigma = \sigma_{yP} / FS \qquad (3.13)$$

where

$\sigma =$ Working stress

$FS =$ Factor of safety

After substituting the working stress, σ , for the equivalent stress, S, equation (3.13) is equal to

$$\frac{\sigma_{yP}}{FS} = \sqrt{\left(\sigma_{av} + \frac{K\sigma_{yP}}{\sigma_c}\sigma_r\right)^2 + 3\left(\tau_{av} + K_t\frac{\sigma_{yP}}{\sigma_e}\sigma_r\right)^2} \qquad (3.14)$$

The maximum shear stress failure theory is also used for predicting failure for ductile materials, although it is less accurate than the distorsion energy theory. The first step is to find the maximum shear stress for a given loading. For two plane loading, the maximum shear stress is

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (3.15)$$

For shafting, this reduces to

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \qquad (3.16)$$

After substituting equations (3.8) and (3.9) into equation (3.16), the maximum shear stress theory predicts failure as,

$$\tau_{max} = \sqrt{\frac{1}{4}\left(\sigma_{av} + \frac{K\sigma_{yP}}{\sigma_e}\sigma_r\right)^2 + \left(\tau_{av} + K_t\frac{\sigma_{yP}}{\sigma_e}\tau_r\right)^2} \qquad \ldots \quad (3.17)$$

For pure shear loading, the working stress is

$$\tau_{max} = \tau_{yP} / FS \qquad (3.18)$$

where,

τ_{yP} = Shear yield stress

FS = Factor of safety

The shear yield stress for uniaxial loading is

$$\tau_{yP} = \sigma_{yP}/2 \qquad (3.19)$$

By substituting equation (3.19) into equation (3.18), t' working stress becomes

$$\tau_{max} = \frac{0.5\sigma_{yP}}{F\ S} \qquad (3.20)$$

Finally, after substituting the equation (3.20) into equation (3.17), equation (3.20) becomes

$$\frac{0.5\ \sigma_{yP}}{FS} = \sqrt{\frac{1}{4}(\sigma_{av} + \frac{K\sigma_{yP}}{\sigma_e}\sigma_r)^2 + (\tau_{av} + K_t\frac{\sigma_{yP}}{\sigma_e}\tau_r)^2} \qquad \ldots \qquad (3.21)$$

Depending on which type of failure theory is to be used the factor of safety can be determined from equation (3.14) or equation (3.21). For both failure theories, it is assumed the normal and shear stress maximums occur simultaneously, which is not necessarily true.

3.6 Consideration of Fatigue Strength Under Fluctuating Stresses:

Machine members are often found to have failed under the action of repeated or fluctuating stresses, though the analysis reveals that the actual maximum stresses were below the ultimate strength of the material and below yield strength. This type of failure occur because stresses are repeated a very large number of times and this failure is called a fatigue failure. Fatigue failures mainly occur at change in cross-section, keyway,or a hole.

When a fatigue strength is to be considered in design and analysis of rotating shaft the modified endurance limit of that shaft is calculated [10], as given in equation.

$$S_e = K_a K_b K_c S'_e \qquad (3.22)$$

where,

S_e = modified endurance limit of mechanical element

S'_e = endurance limit of rotating beam specimen

K_a = surface factor

K_b = size factor

K_c = modifying factor for stress concentration

By modified endurance limit of a material is usually meant that the maximum stress which can be reversed on large number of times without producing fracture. For steels the endurance limit of rotating beam specimen is calculated by equation,

$$S'_e = 0.5\ S_{ut} \tag{3.23}$$

where,

S_{ut} = ultimate tensile strength.

The factors K_a , K_b and K_c are governed by equation surface finish factor is given in [10] as follows:

$$\left.\begin{aligned} K_a &= 0.73 \quad \text{for ground surface finish} \\ K_a &= 0.84 \quad \text{for polished surface finish} \end{aligned}\right\} \tag{3.24}$$

Size factor is taken into account since when alternating stresses are present in bending and torsion the diameter of shaft has also some effect on fatigue failure [2,10], it is given as,

$$K_b = 1.189\ d^{-0.097} \tag{3.25}$$

where,

d = diameter of shaft in mm

modified factor for stress concentration is given as

$$K_c = \frac{1}{K_f} \tag{3.26}$$

where

K_f = fatigue stress concentration factor which is given by equation (3.27)

$$K_f = 1 + q\,(K_t - 1) \qquad (3.27)$$

where,

q = notch sensitivity

K_t = theoretical stress concentration factor

notch sensitivity q [13,16] can be found from Fig. (3.6).

When this modified endurance limit is calculated from eq.(3.22), this modified endurance limit is used in failure theories discussed in Section 3.5 when fatigue strength effect is to be considered.

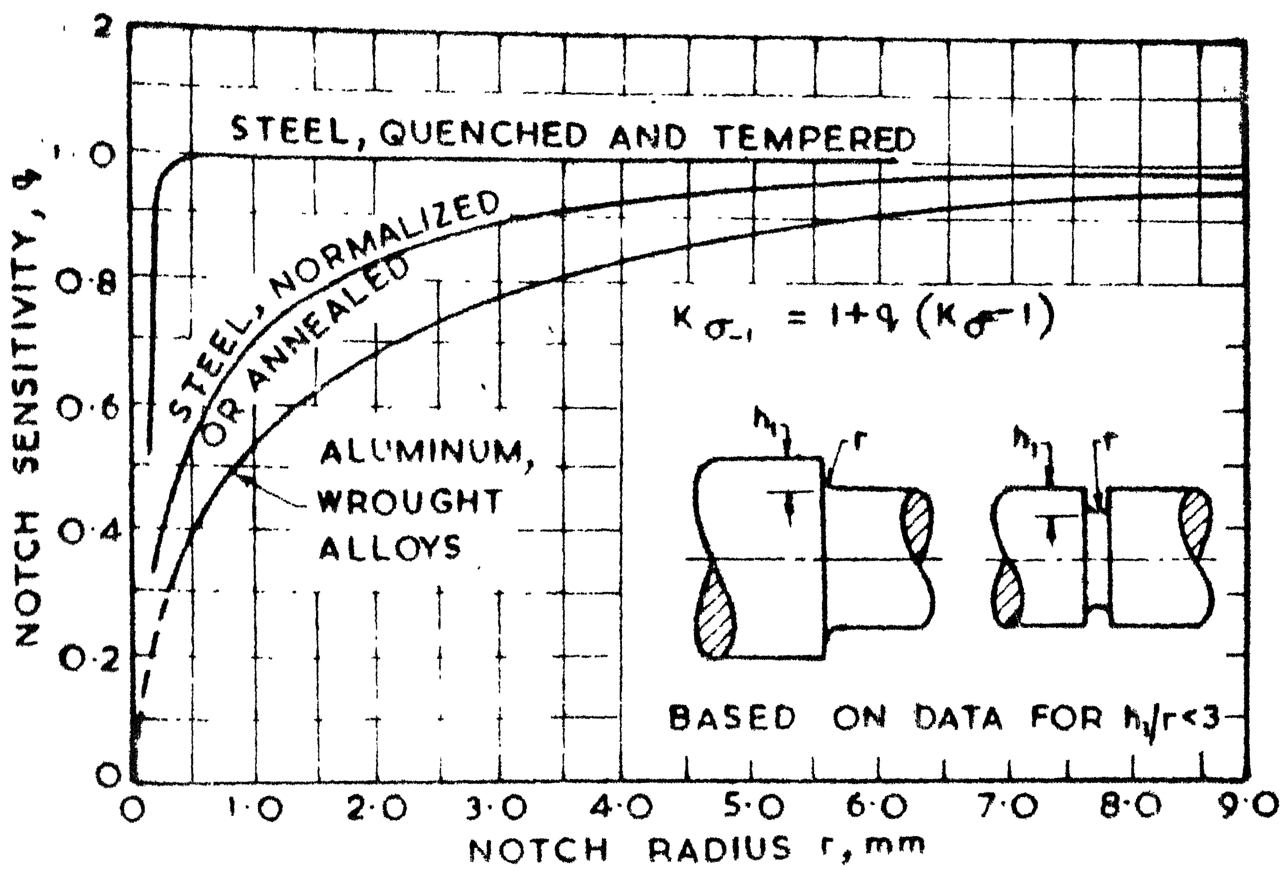


Fig. 3.6 Notch sensitivity, q Ref. [16].

CHAPTER - 4

DESIGN AND ANALYSIS OF MULTI-SPEED GEAR BOX SHAFTS

4.1 Introduction:

This chapter gives a brief idea about, how to analyse gear box shafts. As discussed in Chapters 2 and 3, to analyse any shaft, loading on the shaft should be known. Since gear box provide for a wide range of cutting speeds and torques from a constant speed power input to get proper speeds and torques at the spindle. The loading on these shafts are mainly torques, tangential and radial point loads when the gears on these shafts mesh with gears on other shafts. Bending moments and shear forces on these shafts due to the tangential and radial point loads can be calculated. Then these torques, bending moments and shear forces on the shafts are taken for the analysis of shafts. As discussed in Chapter 3 , using these torques, bending moments and shear forces, factor of safety at any point along the shaft is calculated. Maximum deflection of the shaft is calculated since this is also a factor used to analyse a shaft.

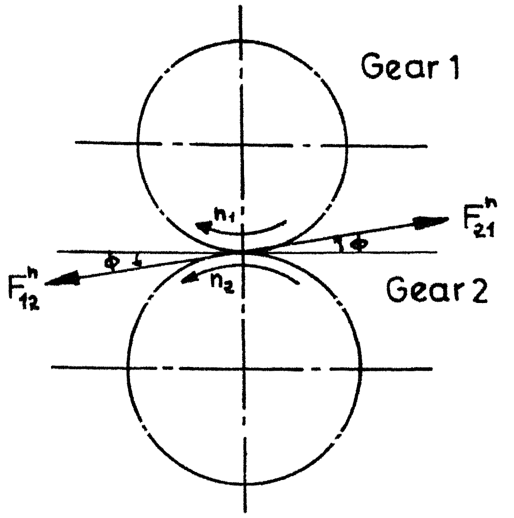
4.2 Force Analysis:

Since multi-speed gear box gives a wide range of speeds and torques at spindle, the main loadings on the intermediate shafts are torques, tangential and radial loads. In this present work the gear box consisting of only spur gears (which are most common), having same module are considered. However, since there is a provision to consider axial loading in the program developed, the gear box shafts having worm or other gears can also be analysed by adding a new subroutine for their force analysis. When two gears mesh each other, forces are exerted on each gear along the pressure angle ϕ, where the two pitch circles of these gears meet as shown in Fig. 4.1(a). Considering free body diagram Fig. 4.1(b) it can be seen that the force on the pitch circle of gear can be replaced by a torque and a point load called normal force on the shaft. Thus proceeding this way we can find the torque and point loads on the shaft at different places where the gears on this shaft meshes with the gears on other shafts.

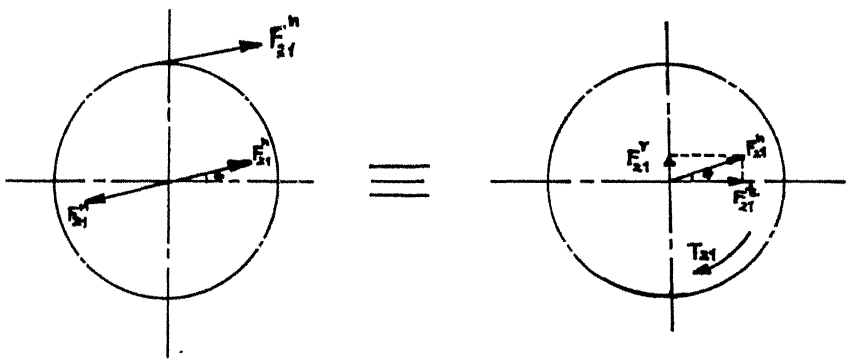
In Fig. (4.1) the forces are defined as

F^n_{21} - normal force exerted by gear 2 against gear 1

F^n_{12} - normal force exerted by gear 1 against gear ?



a) Forces on the Gears,



b) Free-body diagrams of Gear 2,

Fig. 4.1 Forces on spur gears in a gear train.

F^{t}_{21} - tangential force on shaft (in horizontal plane)

F^{r}_{21} - radial force on shaft (in vertical plane)

T_{21} - torque on gear 2.

When these tangential and radial forces on the shaft are calculated, the bending moments and shear forces in horizontal and vertical planes are calculated, then these are combined together and considering theories of failure as discussed in Chapter 3, the factor of safety is calculated. It is seen that, when the shafts in gear box are not coplaner, then also the values of resultant bending moments and shear forces on the shafts donot change. This is due to the fact that, the tangential and radial forces in two planes found at any location are governed by

$$F^{n}_{21} = \sqrt{(F^{t}_{21})^2 + (F^{r}_{21})^2} \qquad (4.1)$$

Similarly the resultant bending moments and shear forces are governed by

$$(BM)_R = \sqrt{(BM)^2_H + (BM)^2_V} \qquad (4.2)$$

and

$$(SF)_R = \sqrt{(SF)^2_H + (SF)^2_V} \qquad (4.3)$$

where,

$(BM)_R$ - resultant bending moment.

$(BM)_H$ - bending moment in horizontal plane.

$(BM)_V$ - bending moment in vertical plane.

$(SF)_R$ - resultant shear force.

$(SF)_H$ - shear force in horizontal plane.

$(SF)_V$ - shear force in vertical plane.

Thus for a compact gear box the shafts can be noncoplaner. This can be seen from Fig. (4.2). In the program developed the equations used to calculate the forces in horizontal and vertical planes from normal force on the gears are as

$$F_H = F^n \cos (110° - \theta) \qquad (4.4)$$

$$F_V = F^n \sin (110° - \theta) \qquad (4.5)$$

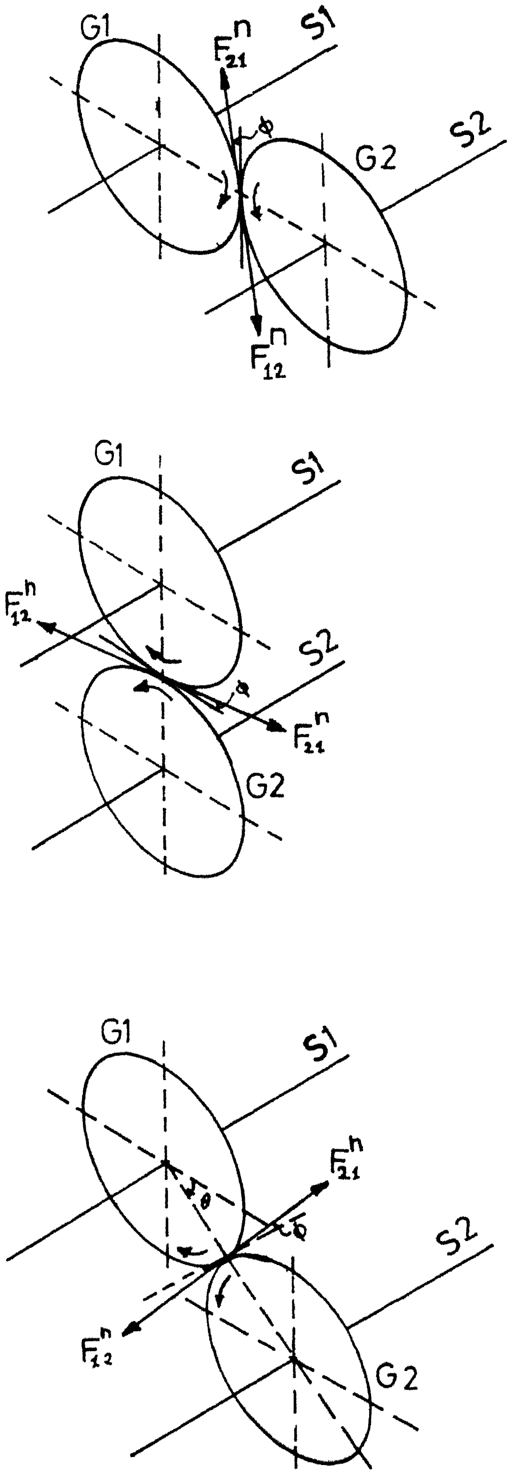
where,

θ - angle between two shafts with respect to horizontal plane as shown in Fig. (4.2)(c).

The angle 110° in equations (4.4) and (4.5) is the addition of pressure angle and the angle between the tangent to pitch circles and line joining the center of the gears as shown in Fig. (4.2)

G1- GEAR 1
G2 - GEAR 2

S1 - SHAFT 1
S2- SHAFT 2



a) $\theta = 0°$

b) $\theta = 90°$

c) $\theta = \theta°$

Fig.4.2 Isometric views of gear arrangement

4.3 Design and Analysis of Gear Box Shafts:

Before designing the gear box shafts the input horse power and speed of the motor should be known. By using the method as described in [15] the gear box is designed, from which the inputs to design and analyse the gear box shafts taken are number of shafts, number of gears on each shaft, number of teeth on each gear, module, pressure angle and speed ratio. In the present developed program it was not possible to get automatically the different possible arrangement of the gears and the sequence of the gears in which they mesh to get possible different output speeds in a particular arrangement. Thus these sequence of the gears which mesh to give proper output speeds is also one of the inputs and this can be done for number of possible different arrangement of the gears. Some rules are to be followed to get a proper design and analysis of gear box shafts, and also to give proper inputs to the program such as the different possible gear arrangements and the placement of the gears in a particular gear arrangement. These rules are as follows,[3]

a) One set of gears must be completely disengaged before the other set begins to mesh.

b) The sum of the teeth of mating gears in a given stage must be the same for same module in a

clustered set since the center distance must remain same.

c) The minimum axial gap between two adjacent gears must be at least two gear widths.

d) The clearance between the fixed gear and sliding gear in the small and medium machine tool, gear box should be 1.5 to 3 mm.

The diameter of a gear box shaft is calculated roughly by using eq. (4.6), [3]

$$d_{(I)} = \sqrt[3]{T_{(I)} \frac{16}{\pi \sigma_s}} \qquad (4.6)$$

where

$d_{(I)}$ = diameter of I^{th} shaft

$T_{(I)}$ = maximum torque on the I^{th} shaft

σ_s = permissible shear stress.

This rough diameter of a shaft is standardised from a table 4.1 [16].

Then if the keyways are present their locations and the steps on the shafts on which the gears are mounted can be given as input if necessary. After giving the bearing positions, the shaft analysis starts. For the shaft analysis as discussed in Section (4.2) the forces and torques are calculated then these loads are used to calculate bending moments and shear forces

TABLE 4.1

Table to select the standard diameter of the shaft

Standard Diameter in cms.	Standard Diameter in cms.	Standard Diameter in cms.	Standard Diameter in cms.	Standard Diameter in cms.
0.6	0.7	0.8	0.9	1.0
1.1	1.2	1.4	1.5	1.6
1.7	1.8	2.0	2.2	2.4
2.5	2.6	2.8	3.0	3.2
3.5	3.6	3.8	4.0	4.2
4.5	4.8	5.0	5.2	5.5
5.6	5.8	6.0	6.2	6.5
6.8	7.0	7.2	7.5	7.8
8.0	8.5	9.0	9.5	10.0

by using transfer matrix as discussed in Chapter 2 . Using any failure theories as discussed in Chapter 3 . the factor of safety and the maximum deflection at any point for a particular gear sequence which has maximum torque and bending moment in particular gear arrangement are calculated. Thus a shaft having least length and safest one is chosen as the best shaft out of the possible options given.

CHAPTER-5

PROGRAMMING CONSIDERATIONS AND EXAMPLES

5.1 Computer Aided Interactive Design and Analysis:

Computer aided design is a technique in which man and machine are blended into a problem - solving team, coupling the best characteristics of each. The result of this combination works better than either man or machine would work alone. [17]

Computer aided design and analysis consists of five stages as, design logic, equations and computations, design checking, engineering paper work generation and graphical layout of design.

In design logic the computer's decision making capability can be utilized in selecting the best of a number of possible design alternatives. In the present work the program can select the best design after analysing each alternative designs.

Almost all the equations and computations can be put in a form suitable for solution. In the present work the transfer matrix method is programmed for iterative calculations, for analysis of shafts as discussed

in chapter 2. Design checking can be a time consuming manual task, especially if numerous critical calculations are involved. Thus the program developed here takes care of this basic function, with appropriate data substitutions for specific design checks, and thus after analysing it selects the best of all.

The computer is ideally suited for use in generating much of the engineering paper work necessary to the successful implementation of a design.

Graphical layout of the designed product can be seen on graphics terminals. The graphical plots of designed and analysed product can also be seen and from which the designer can decide whether the design is safe or not.

The term interactive design and analysis means, the designer has some control over the intermediate results as well as final results, while the program is in execution mode. Designer is provided with the facility of selecting the required options, and/or sometimes allowed to change the values of parameters calculated. All this logic is so developed that the program will automatically take care of the changes to be made according to the new information given by user. Thus interactiveness of the program is a very powerful tool.

The present program has been written in Fortran-10, developed and tested on DEC-1090 system. In the present chapter the major programming considerations are put in the form of flow charts, also the purpose of different subroutines is explained.

5.2 The Main Program:

The purpose of main program is to accept the input data interactively to calculate some of the parameters in itself and others by calling subroutines and lastly to print the results. Input data is stored in one file, which contains answers for the interactive questions which are variables, or single dimensional arrays. The output of the program is stored is one separate file which can be printed to get a permanent record and the same file can be used as input to the graphics program.

The program is written in a very general way so that it can handle number of different cases. Most of the interaction dialoques are built in the main program. This program developed can analyse shafts having any type of loadings and circular cross-section. It can also design and analyse the gear box shafts calculating the forces and torques on it, where input parameters are only its constructional, gears and input power.

While developing the program it is kept in mind that the memory locations are as less as possible. Some 3 D matrices are used in the program these are very limited and the 3rd dimension of these matrices is to lable, just to remember these matrices.

Separate subroutines are written for purposes described here. Flow charts for some subroutines are given.

1. Subroutine MATMUT

   Purpose - To multiply the 2-D matrices

2. Subroutine AMAT

   Purpose - To create an identity matrix

3. Subroutine APINS

   Purpose - To create an in-span point matrix

4. Subroutine - MAT3D

   Purpose - To multiply the 3-D matrices

5. Subroutine MAT23D

   Purpose - To multiply the 3-D and 2-D matrices

6. Subroutine FORC

   Purpose - To calculate the forces on shafts in a gear box, when given gear box constructional parameters and input power and sequence of gears as input

7. Subroutine ANLYS

Purpose - To analyse the shaft, calculation of stress concentrations at any change in cross-section and at keyways, consideration of fatigue failure is included in this subroutine. Finally using all these values and forces on the shaft considering distorsion energy or maximum shear stress failure theories, normal stresses and factor of safeties are calculated along the shaft.

5.3 Graphics Program:

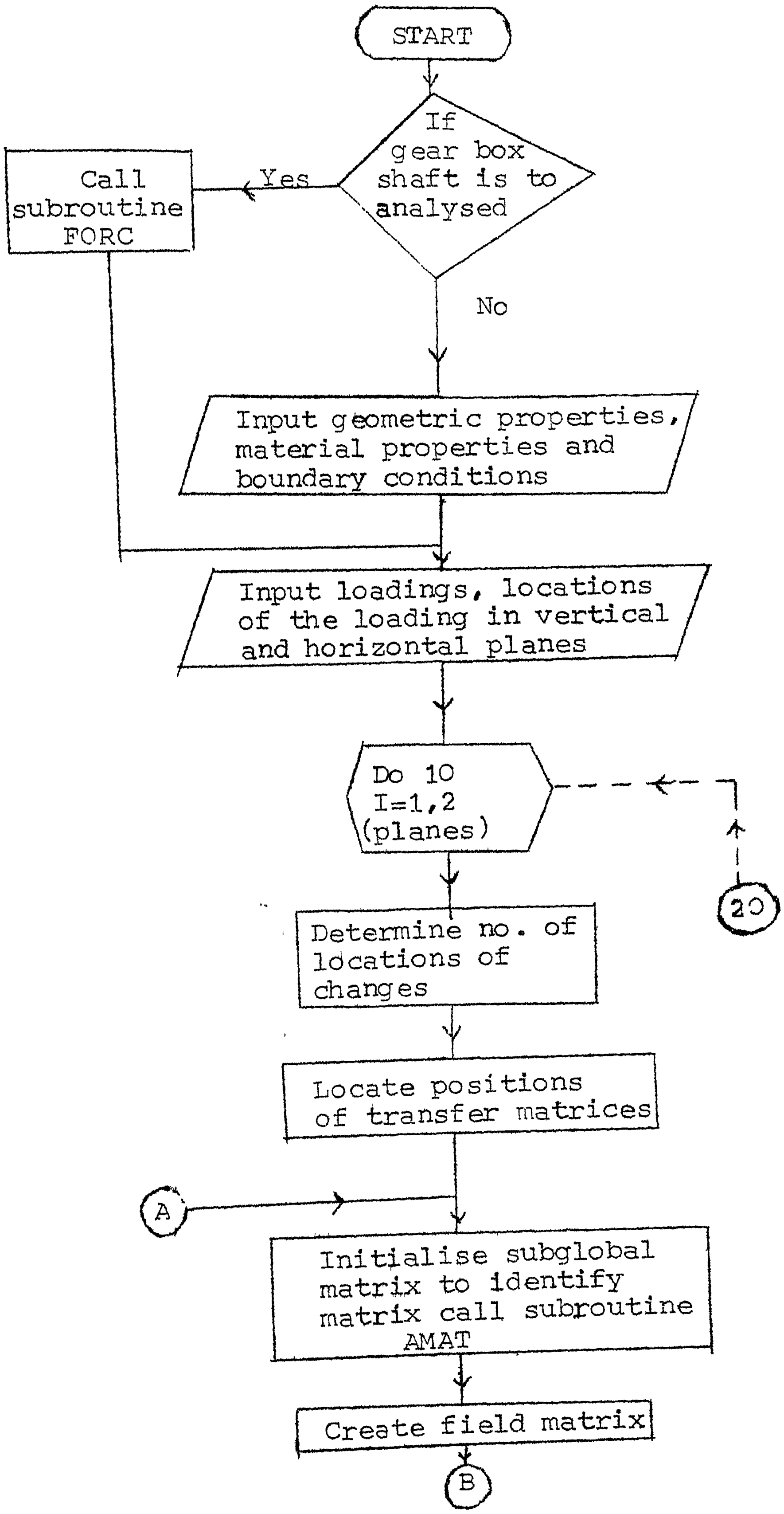
The graphics program is developed by using General Purpose Graphics System manual [20] and [19]. Once the main program is executed the output of this program is used as input to the graphics program. The purpose of this program is to draw the deflection, slope, bending moment, shear force, stress distribution and factor of safety plots along the shaft. Since from these plots the designer will have quick idea about how safe the shaft is.

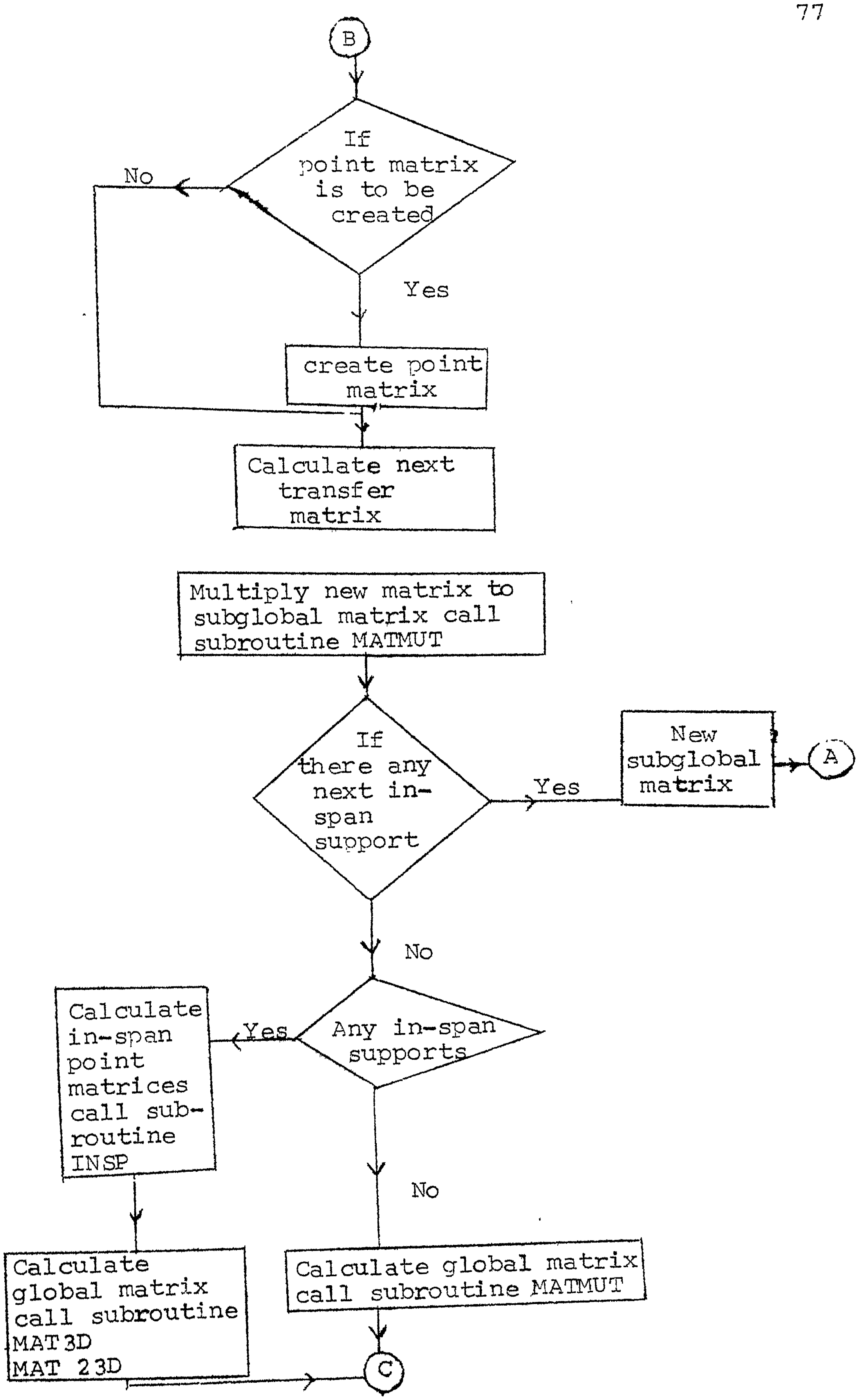
The interactiveness of the program can help the designer to select any plot which he wants to see.

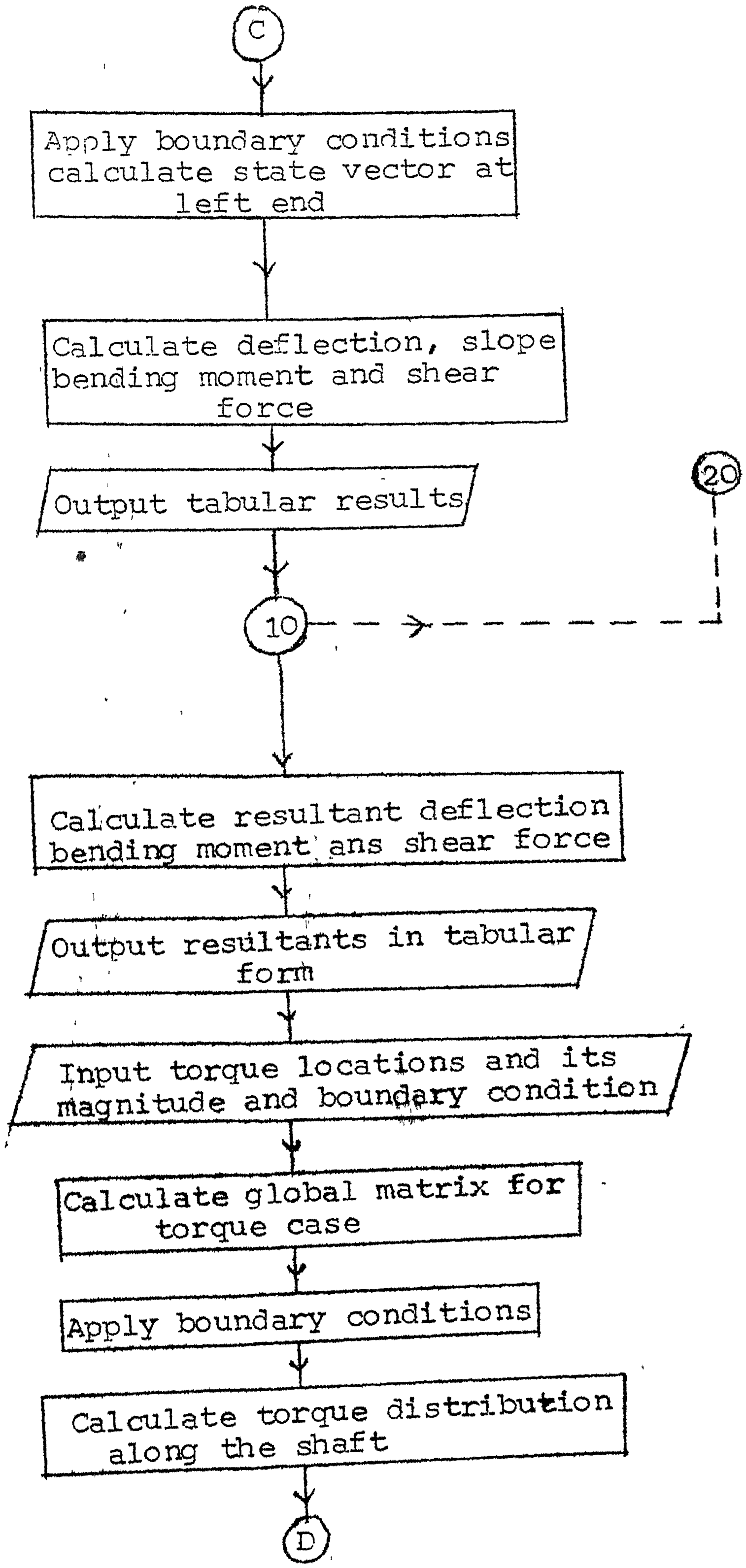
There is also a provision to draw all these plots on the same screen. When shaft selection for particular purpose is to be made then these plots help the designer to quite an extent. The problems like giving window for each plot as input, etc. are overcome in the present developed graphics program. Since this program automatically scales the screen accordingly once by reading the input, by calculating the maximum and minimum of the values.

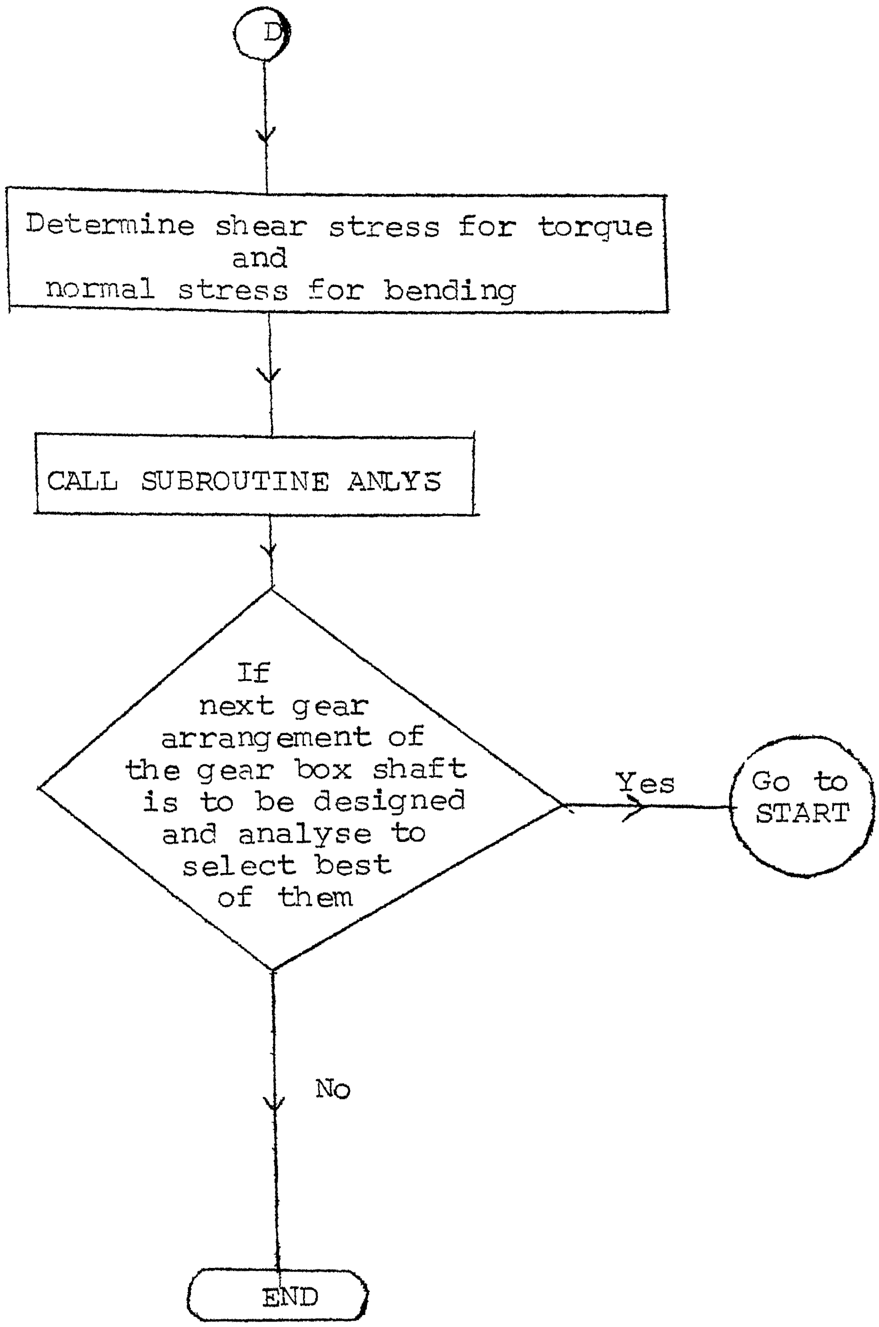
The flow chart of the graphics program is shown here.

MAIN PROGRAM

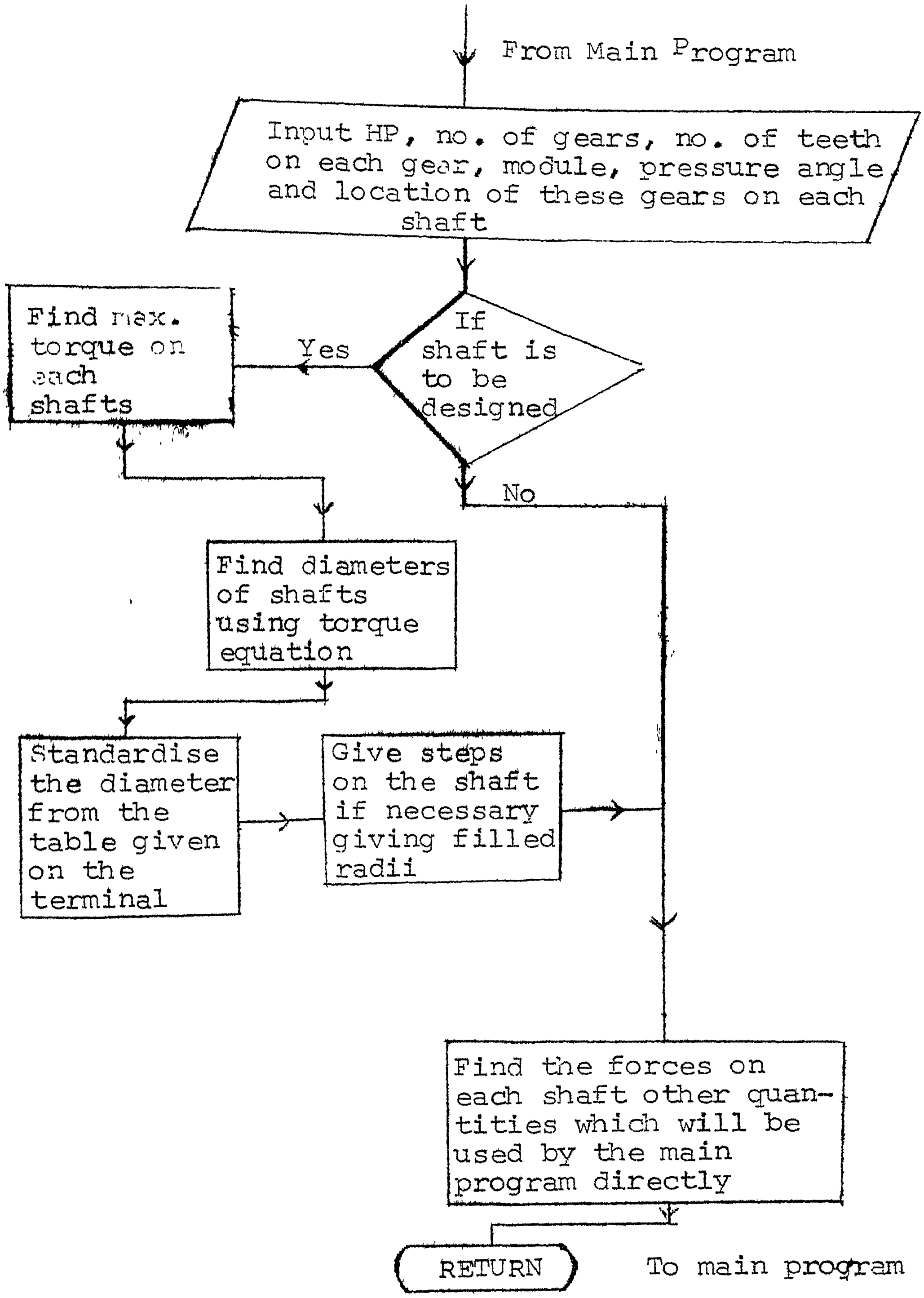



B
If point matrix is to be created
No
Yes
create point matrix
Calculate next transfer matrix
Multiply new matrix to subglobal matrix call subroutine MATMUT
If there any next in-span support
Yes
New subglobal matrix
A
No
Any in-span supports
Yes
Calculate in-span point matrices call sub-routine INSP
No
Calculate global matrix call subroutine MAT3D MAT 23D
Calculate global matrix call subroutine MATMUT
C

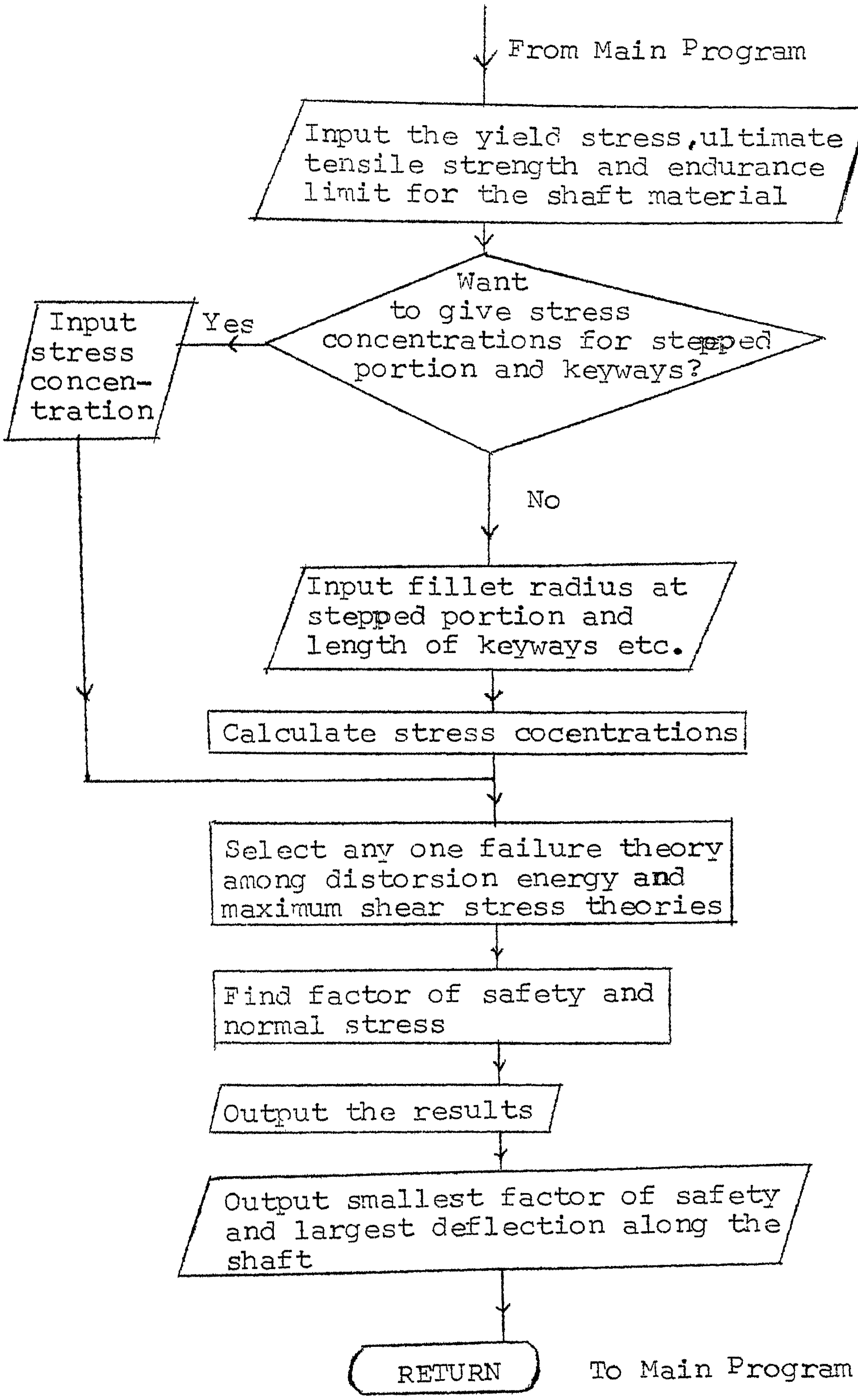

C
Apply boundary conditions calculate state vector at left end
Calculate deflection, slope bending moment and shear force
Output tabular results
20
10
Calculate resultant deflection bending moment ans shear force
Output resultants in tabular form
Input torque locations and its magnitude and boundary condition
Calculate global matrix for torque case
Apply boundary conditions
Calculate torque distribution along the shaft
D


D
Determine shear stress for torque and normal stress for bending
CALL SUBROUTINE ANLYS
If next gear arrangement of the gear box shaft is to be designed and analyse to select best of them
Yes
Go to START
No
END

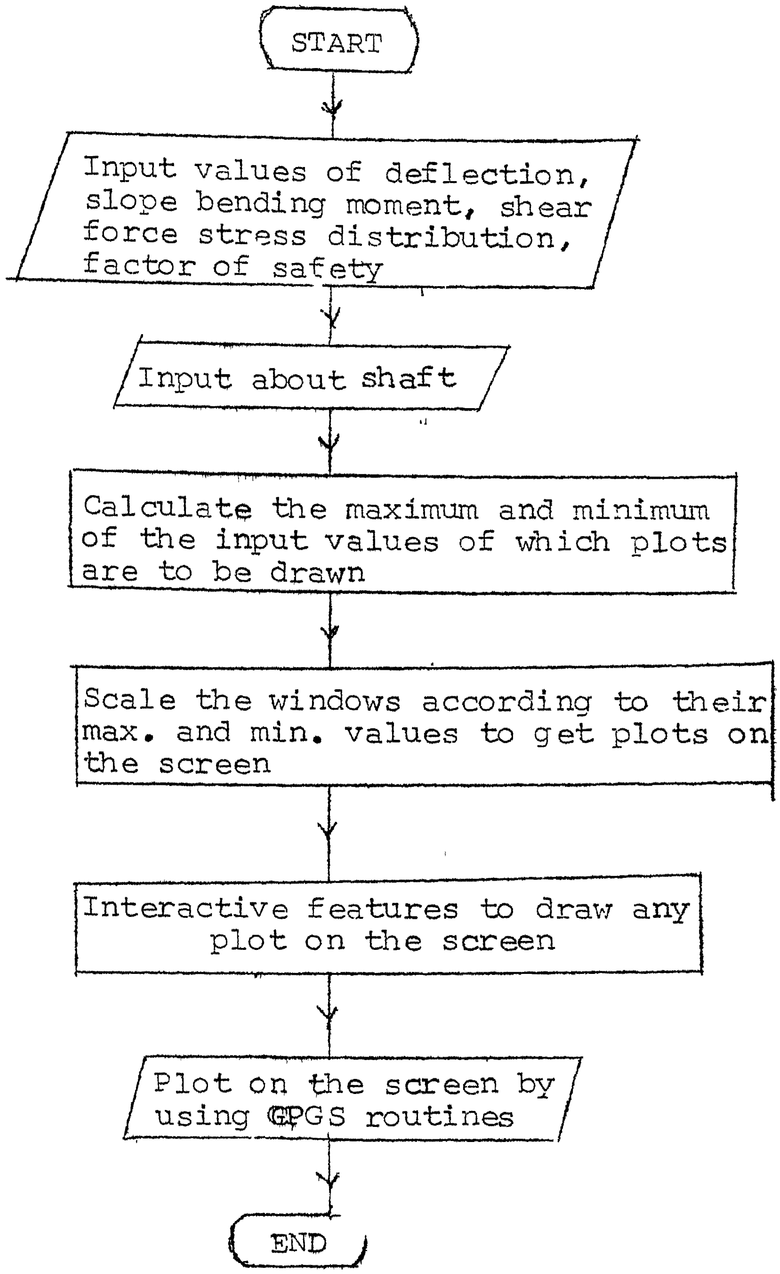
SUBROUTINE FORC



SUBROUTINE ANLYS



GRAPHICS PROGRAM



5.4 Example 1:

This is a analysis problem. The aim of this problem is to get normal stresses and factor of safeties along the shaft. The geometric properties and the loadings are as shown in Fig. (5.1), all the unit in this problem are British units.

The inputs for this example at the time of the execution of the program are given on the terminal as,

DO YOU WANT TO DESIGN AND ANALYSE GEAR BOX SHAFTS? TYPE Y/N : N

DO YOU WANT TO DESIGN A SHAFT? TYPE Y/N : N

PLEASE TYPE IN THE VALUES FOR YOUNG'S MODULUS AND SHEAR MODULUS: 30000000.0, 11000000.0

For boundary conditions like free end type 0, for pinned end type 1, and for fixed end type 2.

PLEASE TYPE IN THE LEFT AND RIGHT END BOUNDARY CONDITIONS AND NUMBER OF SUBGLOBAL MATRICES TO BE FORMED : 1, 0, 2

Type in the geometric properties and loadings for Horizontal plane.

PLEASE TYPE IN NUMBER OF FIELD MATRICES IN EACH SUBGLOBAL MATRIX TO BE CREATED : 7, 2

PLEASE TYPE IN NUMBER OF CONCENTRATED LOAD OCCURRANCES IN EACH SUBGLOBAL MATRIX : 4, 0

IS THERE ANY AXIAL FORCE PRESENT? TYPE Y/N : N

PLEASE TYPE IN DIAMETERS OF SHAFT IN EACH FIELD MATRIX: 0.6, 0.6, 0.6, 0.6, 0.8, 0.8, 0.8, 0.8, 0.8

PLEASE TYPE IN SPAN OF EACH FIELD MATRIX : 2.5, 0.5, 2.0, 1.0, 2.0, 3.0, 4.0, 1.0, 1.0

PLEASE TYPE IN MAGNITUDES OF DISTRIBUTED LOADS IN EACH FIELD MATRIX : 0.0, 0.0, 0.0, 100.0, 100.0, 100.0, 100.0, 100.0, 0.0

PLEASE TYPE IN MAGNITUDES OF GRADIANT OF LINEARLY DISTRIBUTED LOAD IN EACH FIELD MATRIX : 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

PLEASE TYPE IN DISTANCES OF CONCENTRATED LOAD OCCURRANCE FROM LEFT END : 2.5, 3.0, 3.0, 11.0

PLEASE TYPE IN THE MAGNITUDES OF SHEAR FORCES : -200.0, -200.0.

PLEASE TYPE IN THE MAGNITUDES OF CONCENTRATED MOMENTS:200.0

PLEASE TYPE IN THE MAGNITUDES OF SPRING CONSTANTS OF TRANSLATIONAL SPRINGS : 5000000.0

In similar way type in the data for Vertical plane.

Input for the Torque are,

Give the boundary conditions, if shaft is fixed as 1, if free or pinned as 0.

PLEASE TYPE IN THE BOUNDARY CONDITION AT LEFT AND RIGHT END : 0, 0

PLEASE TYPE IN THE NUMBER OF TORQUES APPLIED : 3

PLEASE TYPE IN THE MAGNITUDE OF EACH TORQUE : 600.0, 600.0 , -1200.0

PLEASE TYPE IN THE LOCATIONS OF TORQUES FROM LEFT

END : 0.0, 10.0, 15.0

DO YOU WANT TO USE DISTORSION ENERGY FILURE THEORY

TYPE Y/N : Y

PLEASE TYPE IN THE YIELD STRESS AND ENDURANCE LIMIT

OF SHAFT MATERIAL : 130000.0, 120000.0

DO YOU WANT TO GIVE STRESS CONCENTRATION FACTORS FOR

BENDING AN TORQUE? TYPE Y/N : N

PLEASE TYPE IN THE FILLET RADIUS : 0.01

DO YOU WANT TO CONSIDER FATIGUE FAILURE AT THIS FILLET?

TYPE Y/N : Y

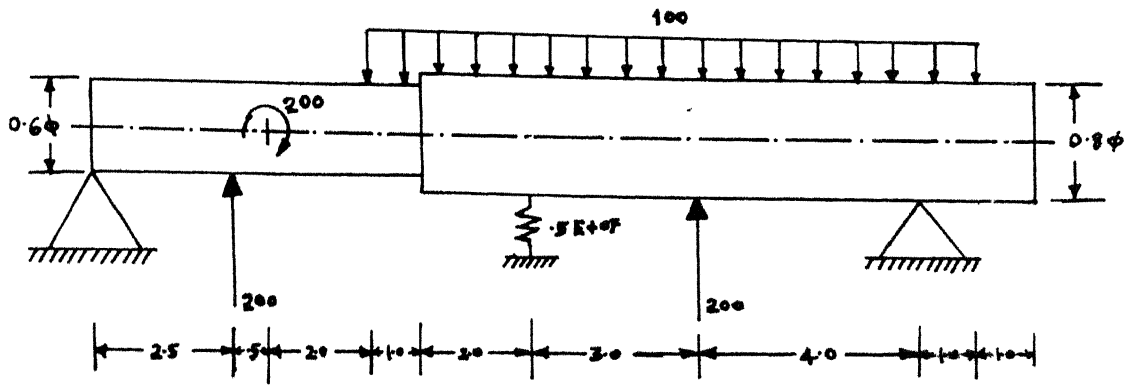
PLEASE TYPE IN THE NOTCH SENSITIVITY : 0.9

PLEASE TYPE IN NUMBER OF KEYWAYS PRESENT, IF NO KEY-WAYS

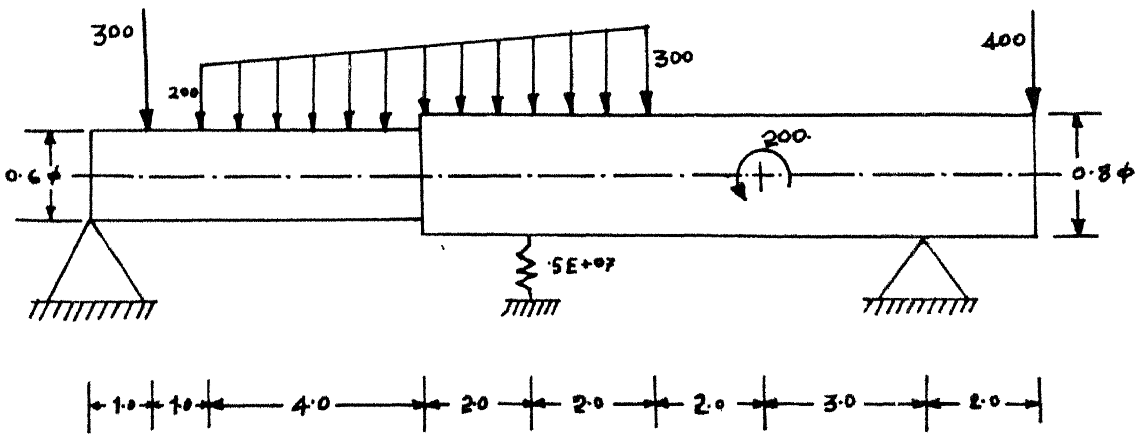
TYPE 0 : 0

The results of the analysis are stored in DATA files, and are given in the following tables (5.1) to (5.4). These tables are used by the graphics program and the plots of the different values can be seen on the graphics terminal.

The smallest factor of safety is 2.3676 at 2.39 inches from left end. The maximum deflection is 0.02916 inches at 3.57 inches from left end.



a) Loading in horizontal plane.



b) Loading in vertical plane.

Fig. 5.1 Loadings for Example 1.

Table 5.1

THESE RESULTS ARE FOR HORIZONTAL PLANE

POSITION FROM LEFT END INCHES	DEFLECTION INCHES	SLOPE ANGLE RADIANS	BENDING MOMENT LB.INCHES	SHEAR FORCE LB.
0.000000	0.000000	0.002113	0.000000	-125.095100
0.340000	-0.000730	0.002074	-43.552335	-125.095100
0.680000	-0.001433	0.001958	-87.134571	-125.095100
1.020000	-0.002083	0.001764	-130.657010	-125.095100
1.360000	-0.002655	0.001492	-174.209340	-125.095100
1.700000	-0.003120	0.001143	-217.761580	-125.095100
2.040000	-0.003454	0.000717	-261.314010	-125.095100
2.380000	-0.003630	0.000212	-304.865340	-125.095100
2.720000	-0.003507	-0.000344	-304.418580	-125.095100
3.060000	-0.003393	-0.000802	-79.971014	71.404840
3.400000	-0.003190	-0.000923	-55.523355	71.404840
3.740000	-0.002754	-0.001000	-31.075590	71.404840
4.080000	-0.002398	-0.001033	-6.628026	71.404840
4.420000	-0.002338	-0.001023	17.819539	71.404840
4.760000	-0.001589	-0.000970	42.267303	71.404840
5.100000	-0.001355	-0.000873	66.714971	71.404840
5.440000	-0.001036	-0.000740	81.432635	51.404843
5.780000	-0.000858	-0.000590	85.190299	27.404843
6.120000	-0.000633	-0.000477	77.337961	-5.095107
6.460000	-0.000532	-0.000439	57.925524	-43.095110
6.800000	-0.000371	-0.000414	26.953287	-71.095110
7.140000	-0.000203	-0.000410	-15.579051	-105.095110
7.480000	-0.000131	-0.000434	-69.671388	-117.095110
7.820000	0.000012	-0.000491	-135.323730	-211.095110
8.160000	0.000177	-0.000578	-135.475140	237.535740
8.500000	0.000416	-0.000633	-60.492989	235.535740
8.840000	0.000647	-0.000619	2.929162	154.535740
9.180000	0.000876	-0.000632	54.791313	135.535740
9.520000	0.001093	-0.000589	95.093464	101.535740
9.860000	0.001239	-0.000527	123.335620	57.535738
10.200000	0.001459	-0.000452	141.017770	33.535736
10.540000	0.001600	-0.000370	146.639420	-0.454262
10.880000	0.001710	-0.000288	149.702070	-34.454262
11.220000	0.001860	-0.000206	167.234220	131.535740
11.560000	0.001865	-0.000100	206.146370	47.535736
11.900000	0.001879	0.000025	233.528520	53.535736
12.240000	0.001851	0.000161	249.350570	29.535736
12.580000	0.001773	0.000304	253.612420	-4.454264
12.920000	0.001644	0.000445	246.314970	-33.454264
13.260000	0.001456	0.000579	227.457120	-72.454264
13.600000	0.001242	0.000699	197.039270	-103.454260
13.940000	0.000978	0.000799	155.061420	-110.454260
14.280000	0.000682	0.000872	101.523570	-174.454260
14.620000	0.000365	0.000911	36.425721	-203.454260
14.960000	0.000039	0.000911	-40.232129	-217.454250
15.300000	-0.000264	0.000890	-24.510693	70.000005
15.640000	-0.000561	0.000881	-6.490591	36.000005
15.980000	-0.000859	0.000880	-0.030690	2.000005
16.320000	-0.001158	0.000880	-0.010688	0.000007
16.660000	-0.001458	0.000880	-0.010585	0.000007
17.000000	-0.001757	0.000880	-0.010683	1.000007

Table 5.2

THESE RESULTS ARE FOR VERTICAL PLANE

POSITION FROM LEFT END INCHES	DEFLECTION INCHES	SLOPE ANGLE RADIANS	BENDING MOMENT LB.INCHES	SHEAR FORCE LB.
0.000000	0.000000	-0.012957	0.000000	551.328920
0.340000	0.003454	-0.012737	190.851830	551.328920
0.680000	0.006743	-0.012277	381.703660	551.328920
1.020000	0.012896	-0.011127	566.555500	551.328920
1.360000	0.015535	-0.010339	555.407330	251.328920
1.700000	0.019471	-0.009092	744.294160	251.328920
2.040000	0.022853	-0.007687	832.350650	253.316920
2.380000	0.025213	-0.006133	907.408500	183.126120
2.720000	0.027355	-0.004408	958.197350	114.350720
3.060000	0.028301	-0.002731	984.825200	12.305827
3.400000	0.028932	-0.000974	986.001050	-33.421050
3.740000	0.028358	0.000707	963.635100	-135.073550
4.080000	0.026392	0.002419	911.834250	-181.711000
4.420000	0.027204	0.004011	839.407600	-253.275640
4.760000	0.025019	0.005421	730.304150	-333.261300
5.100000	0.023518	0.006626	604.712490	-414.733500
5.440000	0.021030	0.007577	453.161350	-500.031800
5.780000	0.018278	0.008225	269.120000	-585.473500
6.120000	0.015363	0.008473	56.190020	-665.761030
6.460000	0.012431	0.008413	-185.001330	-751.933590
6.800000	0.009532	0.008263	-457.363390	-842.571040
7.140000	0.006716	0.007921	-758.981450	-931.795570
7.480000	0.004139	0.007432	-1041.145800	-1022.351500
7.820000	0.001555	0.006686	-1354.350700	-1114.373000
8.160000	-0.000580	0.005793	-1550.016000	593.441150
8.500000	-0.002369	0.004969	-1372.502000	533.941050
8.840000	-0.003901	0.004241	-1217.500000	407.591150
9.180000	-0.005206	0.003501	-1095.501500	303.101051
9.520000	-0.006307	0.003010	-1070.998700	210.551150
9.860000	-0.007221	0.002419	-952.481930	107.831050
10.200000	-0.007759	0.001920	-920.420240	58.004152
10.540000	-0.008514	0.001404	-903.304820	58.004152
10.880000	-0.008906	0.000992	-880.183410	58.004152
11.220000	-0.009124	0.000412	-857.062000	58.004152
11.560000	-0.009178	-0.000064	-833.940590	58.004152
11.900000	-0.009072	-0.000528	-810.819170	58.004152
12.240000	-0.008802	-0.001056	-787.697750	58.004152
12.580000	-0.008313	-0.001608	-764.576340	58.004152
12.920000	-0.007700	-0.002146	-741.454930	58.004152
13.260000	-0.006877	-0.002670	-718.333510	58.004152
13.600000	-0.005877	-0.003181	-695.212100	58.004152
13.940000	-0.004706	-0.003679	-672.090680	58.004152
14.280000	-0.003358	-0.004164	-648.969270	58.004152
14.620000	-0.001857	-0.004636	-625.847850	58.004152
14.960000	-0.000207	-0.005095	-602.726440	58.004152
15.300000	0.001523	-0.005516	-580.006300	400.000000
15.640000	0.003387	-0.005861	-544.006300	400.000000
15.980000	0.005055	-0.006129	-408.006300	400.000000
16.320000	0.007801	-0.006321	-272.006300	400.000000
16.660000	0.009939	-0.006436	-136.006300	400.000000
17.000000	0.012223	-0.006474	-0.006302	400.000000

Table 5.3

THESE RESULTS ARE FOR RESULTANT VALUES

POSITION FROM LEFT END INCHES	RESULTANT DEFLECTION INCHES	RESULTANT BENDING MOMENT LB.INCHES	RESULTANT SHEAR FORCE LB.
0.000000	0.000000	0.000000	575.759070
0.340000	0.003511	195.756080	575.759070
0.680000	0.006933	391.510170	575.759070
1.020000	0.013065	561.426100	291.034510
1.360000	0.016845	678.164920	291.034510
1.700000	0.020217	775.402330	291.034510
2.040000	0.023122	872.978980	283.067110
2.380000	0.025502	957.253200	221.547232
2.720000	0.027305	1006.391600	134.857540
3.060000	0.028504	988.066020	63.427447
3.400000	0.029096	988.302400	76.271447
3.740000	0.029048	961.130640	127.751030
4.080000	0.028493	911.858200	195.420050
4.420000	0.027310	840.090810	209.059570
4.760000	0.025574	734.573240	345.838700
5.100000	0.023556	613.247640	423.284810
5.440000	0.021068	460.724500	501.406180
5.780000	0.018249	262.261700	504.005340
6.120000	0.015375	95.548920	563.961930
6.460000	0.012443	144.021460	750.620750
6.800000	0.009541	356.156910	843.575840
7.140000	0.006721	759.141320	942.505810
7.480000	0.004041	1093.368800	1037.415900
7.820000	0.001565	1460.632900	1134.005500
8.160000	0.000611	1565.818000	644.234070
8.500000	0.002405	1373.634500	543.192340
8.840000	0.003954	1217.503500	441.440690
9.180000	0.005273	1096.871100	336.152350
9.520000	0.006401	1011.476700	233.700150
9.860000	0.007335	966.494300	126.976430
10.200000	0.008091	937.097430	75.823544
10.510000	0.008668	915.124970	68.905747
10.860000	0.009068	891.358460	76.238770
11.220000	0.009300	873.219630	148.075030
11.560000	0.009365	859.042040	118.902420
11.900000	0.009255	843.779170	93.060344
12.240000	0.008994	1018.080730	74.141247
12.580000	0.008530	997.300000	68.150527
12.920000	0.007874	973.143590	78.128517
13.260000	0.007031	946.083070	99.376275
13.600000	0.006007	916.040150	125.329740
13.940000	0.004856	885.766710	156.000150
14.280000	0.003436	855.018010	187.249420
14.620000	0.001902	826.650740	219.275800
14.960000	0.000211	803.734010	251.820340
15.300000	0.001685	680.417000	405.078810
15.640000	0.003531	544.145020	401.016730
15.980000	0.005720	408.000300	400.005000
16.320000	0.007887	272.006340	400.000000
16.660000	0.010165	136.000300	400.000000
17.000000	0.012349	0.012403	400.000000

Table 5.4

RESULTS FOR DISTORTION ENERGY THEORY

POSITION FROM LEFT END INCHES	NORMAL STRESS Lb./Sq.Inch	FACTOR OF SAFETY
0.0000	24513.0000	7.3424
0.3400	28154.1570	6.3934
0.6800	36984.3130	4.6609
1.0200	47874.9250	3.7596
1.3600	53867.3390	3.3415
1.7000	60377.9080	2.9961
2.0400	66135.3350	2.7094
2.3800	72009.4630	2.4947
2.7200	75219.4200	2.3930
3.0600	74062.2510	2.4311
3.4000	71081.0870	2.4297
3.7400	72467.1070	2.4631
4.0800	69197.0160	2.6013
4.4200	64273.1[illegible]	2.8033
4.7600	57764.8130	3.1158
5.1000	49825.1210	3.6125
5.4400	40778.3384	4.4141
5.7800	31615.8310	5.6932
6.1200	15648.1420	11.5033
6.4600	11877.5240	15.1545
6.8000	17157.9990	10.4907
7.1400	21915.0310	7.2242
7.4800	34234.4550	5.2571
7.8200	44568.6310	4.0171
8.1600	47861.8550	3.7668
8.5000	42283.4010	4.2576
8.8400	37775.2950	4.7649
9.1800	34327.2350	5.2436
9.5200	31905.8400	5.6416
9.8600	30473.3800	5.9074
10.2000	34777.2520	5.1758
10.5400	31252.3450	5.2551
10.8800	33689.5000	5.3429
11.2200	33263.9960	5.4113
11.5600	32933.5150	5.4656
11.9000	32560.1590	5.5218
12.2400	36763.5911	4.8962
12.5800	36239.0540	4.9670
12.9200	35647.9490	5.0494
13.2600	34993.2900	5.1438
13.6000	34288.4250	5.2496
13.9400	33558.1[illegible]	5.3638
14.2800	32849.4000	5.4811
14.6200	32187.2860	5.5923
14.9600	31605.4180	5.6843
15.3000	20309.0020	8.6631
15.6400	16239.3030	11.0642
15.9800	12161.0590	14.7770
16.3200	8125.3886	22.1528
16.6600	4075.2385	44.1692
17.0000	367.5519	469.7267

5.5 Example 2:

This is a design and analysis problem. The aim of this problem is to get safe diameter of the shaft for a given loading, considering stress concentrations at keyways and to find normal stresses and factors of safety. The geometric properties and the loadings are as shown in Fig. (5.2). The problem is solved in SI units.

The input is as follows,

DO YOU WANT TO DESIGN AND ANALYSE GEAR BOX SHAFTS?
TYPE Y/N : N
DO YOU WANT TO DESIGN THE SHAFT? TYPE Y/N : Y
PLEASE TYPE IN THE MAXIMUM TORQUE, BENDING MOMENT AND
PERMISSIBLE SHEAR STRESS : 4792.0, 4500.0, 420.0
The calculated diameter of the shaft is 3.8 cm.
DO YOU WANT TO STANDARDISED THE DIAMETER OF THE SHAFT?
TYPE Y/N : Y
The table of standard diameters of shaft (as shown in chapter 4) is shown on the Terminal.
PLEASE TYPE IN THE NEAREST STANDARD DIAMETER IN CMS:4.0
PLEASE TYPE IN THE VALUES FOR YOUNG'S MODULUS AND
SHEAR MODULUS : 21000000.0, 8400000.0
Type in the locations at the loadings and their magnitudes.
PLEASE TYPE IN THE LEFT AND RIGHT END BOUNDARY CONDITION
AND NUMBER SUBGLOBAL MATRICES TO BE FORMED : 0, 0, 3

PLEASE TYPE IN THE NUMBER OF FIELD MATRICES IN EACH SUBGLOBAL MATRIX TO BE CREATED : 1, 1, 1

PLEASE TYPE IN THE NUMBEP OF CONCENTRATED LOAD OCCURRANCES IN EACH SUBGLOBAL MATRIX : 1, 0, 1

IS THERE ANY AXIAL FORCE PRESENT? TYPE Y/N : N

PLEASE TYPE IN SPANS OF EACH FIELD MATRIX : 15.0, 80.0, 25.0

PLEASE TYPE IN THE DISTANCES OF CONCENTRATED LOAD OCCURRANCE FROM LEFT END : 0.0, 120.0

PLEASE TYPE IN THE MAGNITUDES OF SHEAR FORCES : 300.5, 160.3

Inputs for Torque -

PLEASE TYPE IN THE BOUNDARY CONDITIONS AT LEFT AND RIGHT END : 0, 0

PLEASE TYPE IN THE NUMBER OF TORQUES APPLIED : 2

PLEASE TYPE IN THE MAGNITUDES OF EACH TORQUE : 1792.0 -1792.0

PLEASE TYPE IN THE LOCATIONS OF TORQUES FROM LEFT END: 0.0, 120.0

DO YOU WANT TO USE DISTORSION ENERGY FILURE THEORY? TYPE Y/N : Y

PLEASE TYPE IN THE YIELD STRESS AND ENDURANCE LIMIT: 3164.0, 2953.0

PLEASE TYPE IN THE NUMBER OF KEY-WAYS PRESENT : 2

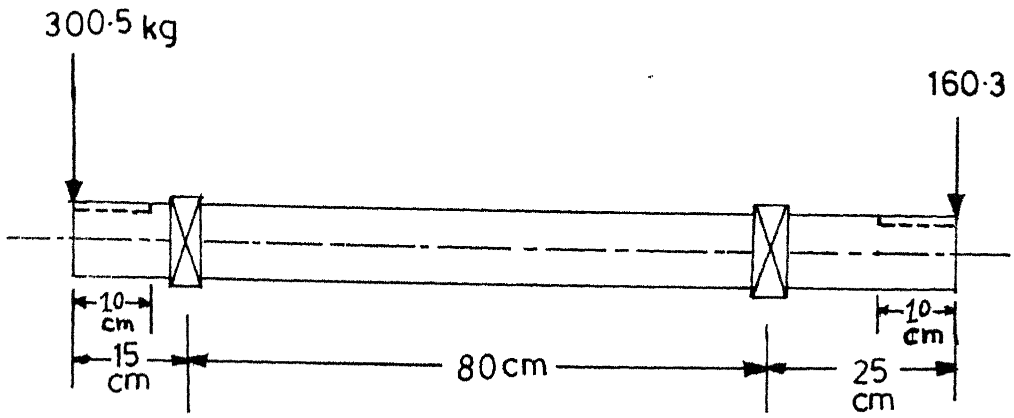


Fig.5.2 Example 2 .

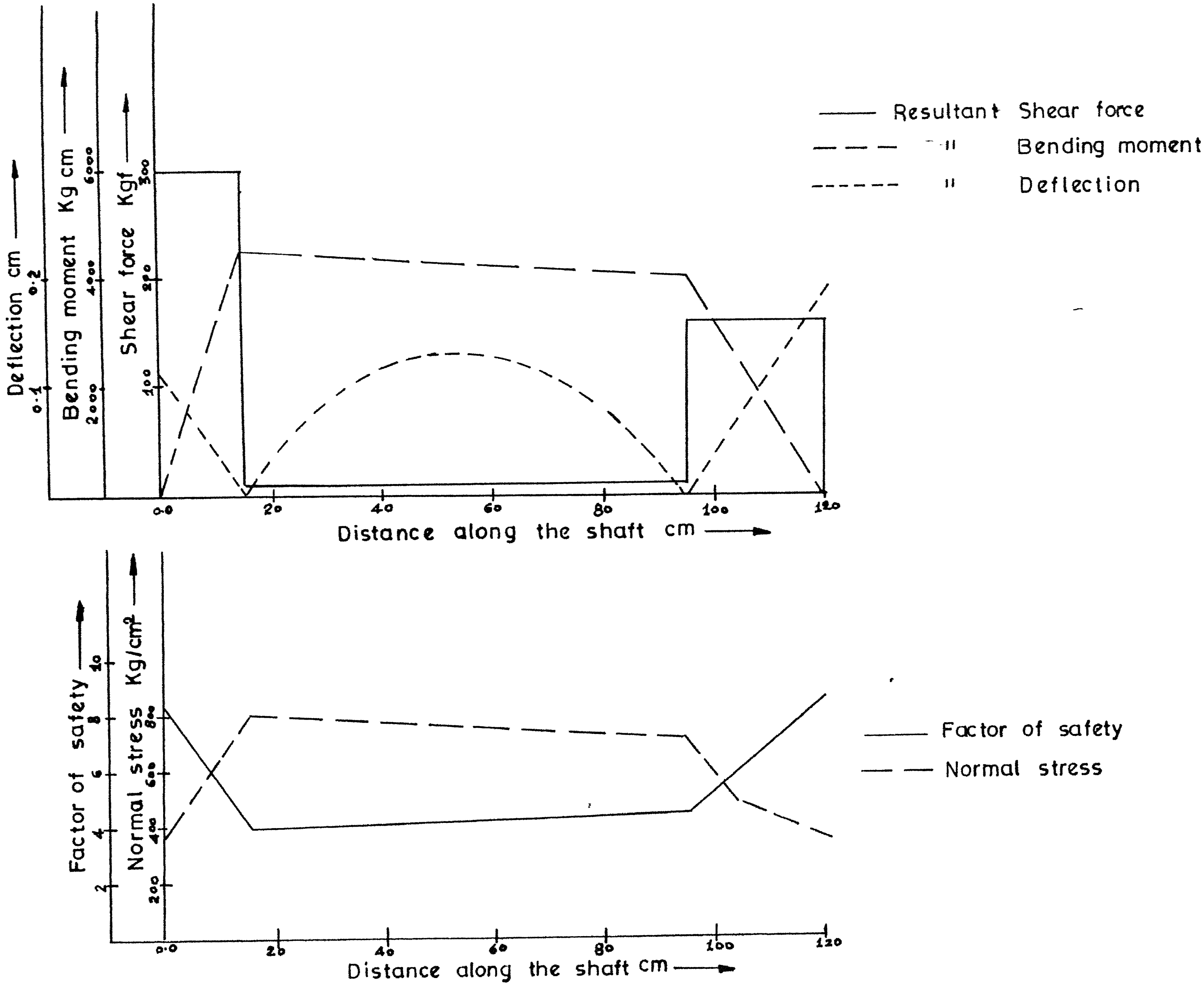


Fig. 5.3 Plots for shaft of Example 2.

PLEASE TYPE IN THE STRESS CONCENTRATION AT EACH
KEY WAY : 1.5, 1.5
PLEASE TYPE IN THE LEFT STARTING POSITION OF EACH
KEY WAY : 0.0, 110.0
PLEASE TYPE IN THE LENGTH OF EACH KEY WAY : 10.0, 10.0

The results of this designed and analysed shaft are stored in the DATA FILES. The plots of the resultant values normal stresses and factores of safety are as shown in the Fig. (5.3).

The diameter of the designed shaft is 4.0 cm. The smallest factor of safety is 3.92 which is at 15.0 cm from left end. The maximum deflection is 0.19 cm at 120.0 cm from left end i.e. at right end.

5.6 Example 3:

This example illustrates a 4-speed machine tool gear box. The problem aims to design and analyse the gear box shafts for a given gear arrangement. This problem is executed for different possible gear arrangements in the gear box. Then a shortest and an appropriate gear box shaft is selected from the different specified options. Two gear arrangements of a 4-speed gear box are shown in Fig. (5.4)a and (5.4)b. The shortest and appropriate gear box shafts are for gear arrangement 2 as in Fig. (5.4)b. The inputs for the gear arrangement 2 (taken from [3]) are as follows:(all inputs are in SI Units)

DO YOU WANT TO DESIGN AND ANALYSE GEAR BOX SHAFTS?

YES Y/N : Y

PLEASE TYPE IN THE MODULE OF THE GEARS IN CMS : 0.3 cm

PLEASE TYPE IN THE PRESSURE ANGLE OF THE GEARS IN DEGREES : 20°

PLEASE TYPE IN THE HORSE POWER OF THE INPUT MOTOR: 10.0

PLEASE TYPE IN THE SPEED OF THE INPUT MOTOR:

PLEASE TYPE IN THE NUMBER OF SHAFTS IN THE GEAR BOX: 3

Give code 1 for the sliding shafts and 0 for non-sliding shafts.

PLEASE TYPE IN THE CODES FOR SLIDING AND NON-SLIDING SHAFTS : 1, 0, 1

PLEASE TYPE IN THE ANGLE AT WHICH THE SECOND TO LAST SHAFTS ARE, WITH THE FIRST SHAFT : 0.0, 0.0, 0.0

PLEASE TYPE IN THE DENSITY, BREADTH OF GEARS : 0.0072, 3.0,

PLEASE TYPE IN YOUNG'S MODULUS AND SHEAR MODULUS FOR SHAFT MATERIAL : 210000000.0, 8400000.0

PLEASE TYPE IN THE NUMBER OF GEARS ON EACH SHAFT: 2, 4, 2

PLEASE TYPE IN THE NUMBER OF TEETH ON EACH GEAR ON EACH SHAFT : 24, 20

PLEASE TYPE IN THE POSITIONS OF THE GEARS FROM LEFT END BEARING ON EACH SHAFT: 2.0, 5.0, 14.0, 17.0

PLEASE TYPE IN THE POSITION OF THE RIGHT END BEARING: 19.0

PLEASE TYPE IN THE SEQUENCE OF THE GEARS FOR PARTICULAR COMBINATION: 1, 1, 2, 1

The inputs for gear arrangement 1, are also given in the similar manner.

The diameters of all the shafts are calculated and then they are standardised from the table typed on the terminal of the time of execution. The steps on the shafts and the lengths of the keyways are also given. Then the analysis of the intermediate non-sliding shafts start.

From the plots of these results of the two arrangements shown in Fig. (5.5)a and (5.5)b it can be seen that, for arrangement 2 the intermediate non-sliding shaft design is safe and the length of the gear box is small.

The diameters of the shafts in gear arrangements 1 and 2 are same, as same maximum torques on each shafts are present, only the lengths of the shafts in gear arrangement 2 are smaller than the gear arrangement 1.

The diameters of the shafts obtained from the design, in gear arrangement 2 are,

diameter of shaft 1 is 2.0 cm.

diameter of shaft 2 is 2.5 cm.

diameter of shaft 3 is 3.0 cm.

Lengths of the shafts in gear arrangement 1 are 25 cm.

Lengths of the shafts in gear arrangement 2 are 19 cm.

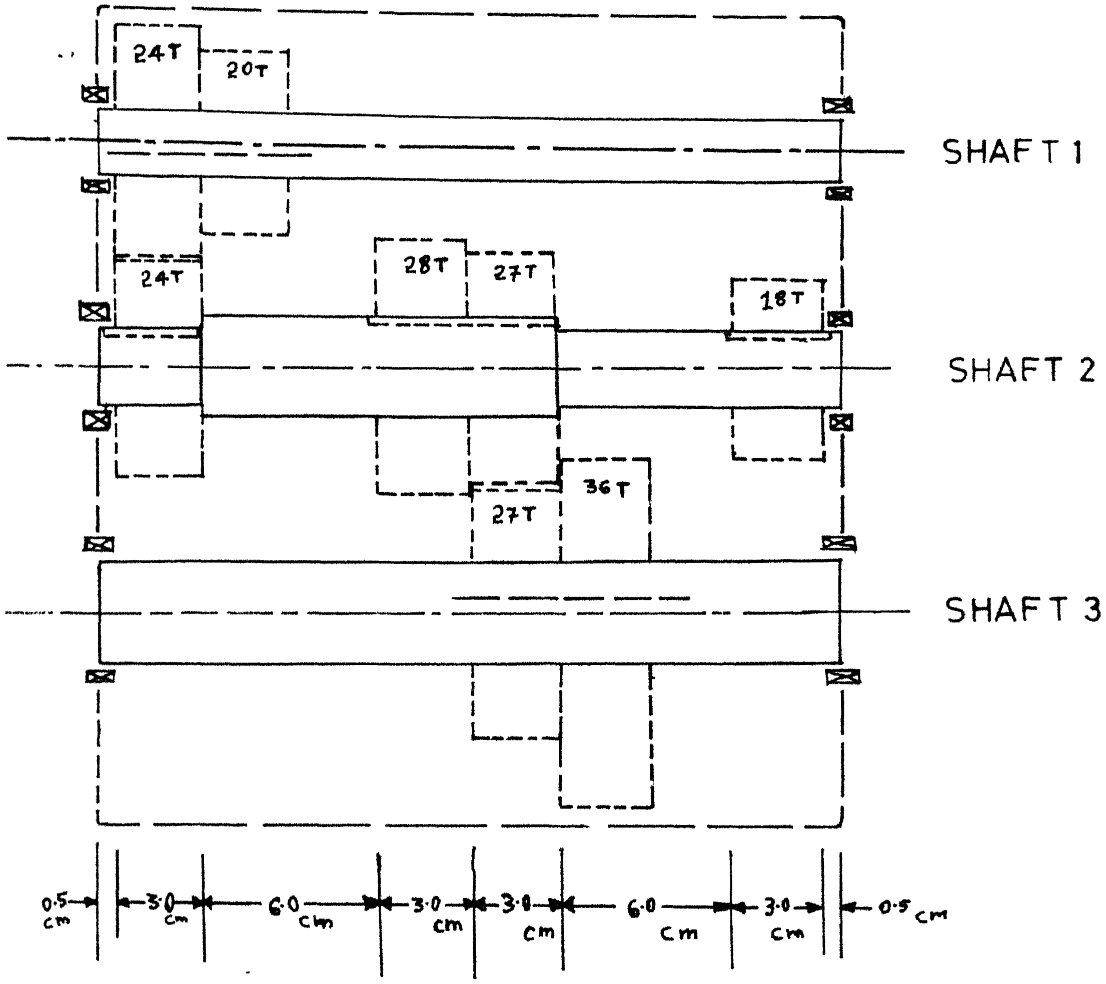


Fig. 5.4 (a) Example 3 ; 4 Speed gear-box 1st arrangement.

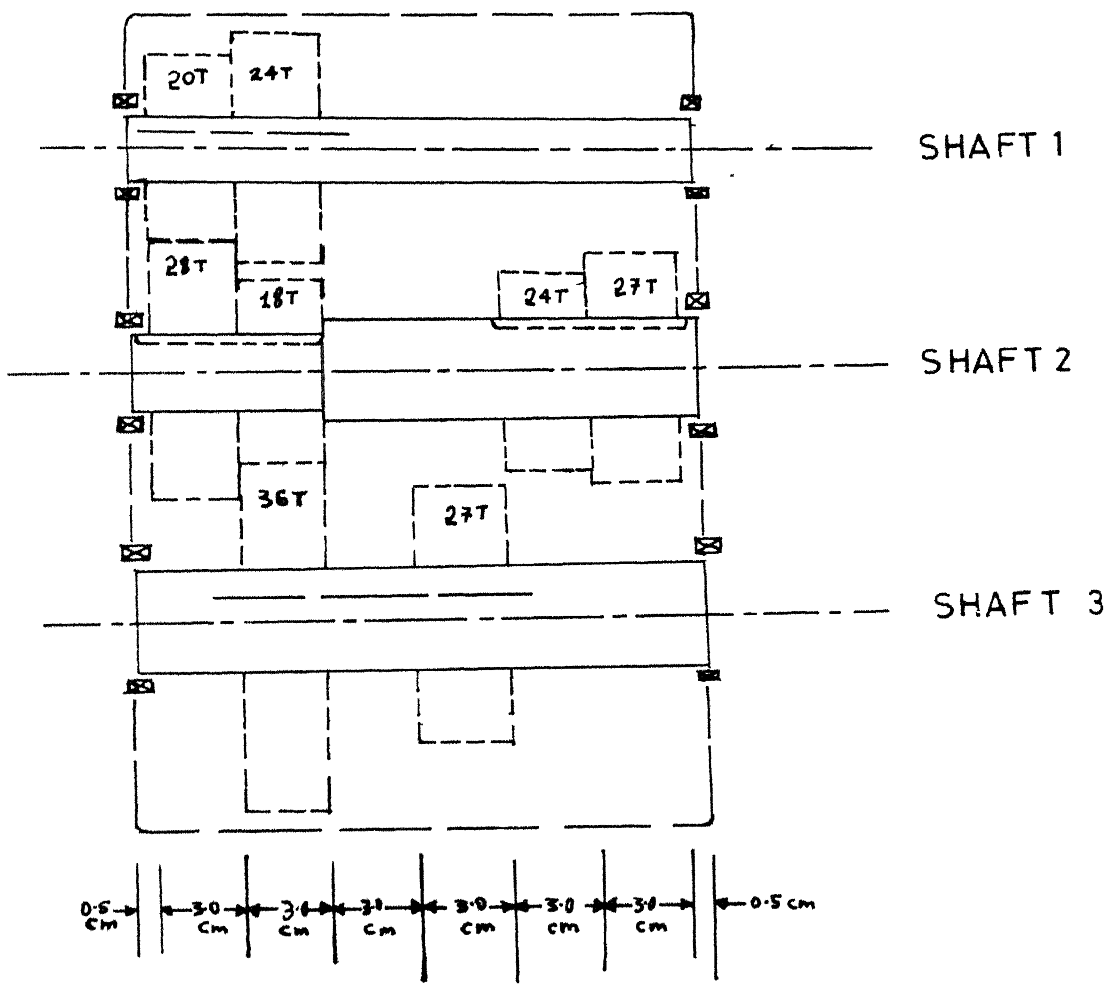


Fig. 5.4 (b) Example 3, 4 Speed gear-box 2nd arrangement.

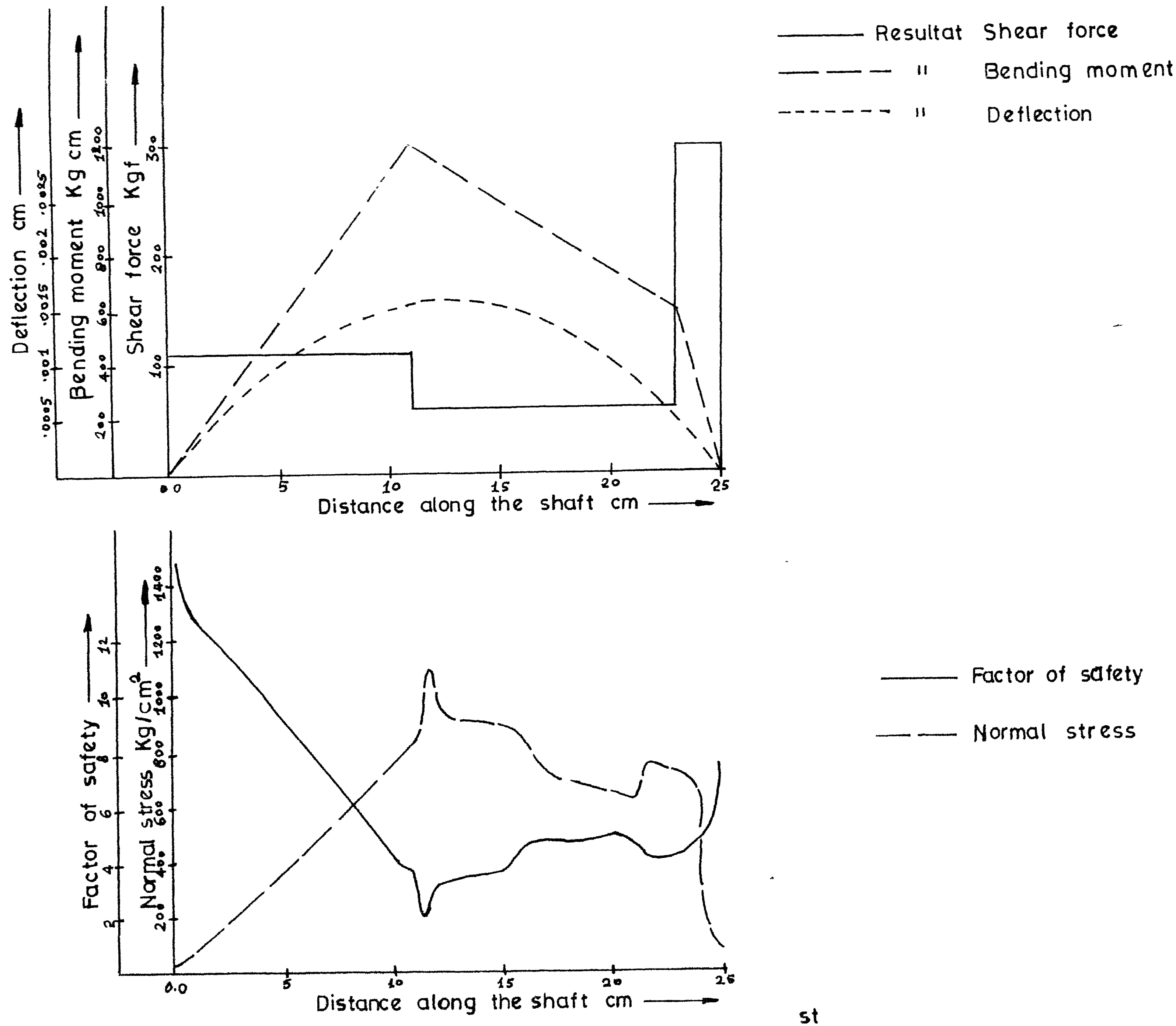


Fig 5.5(a) Plots for Example 3 1st gear arrangement

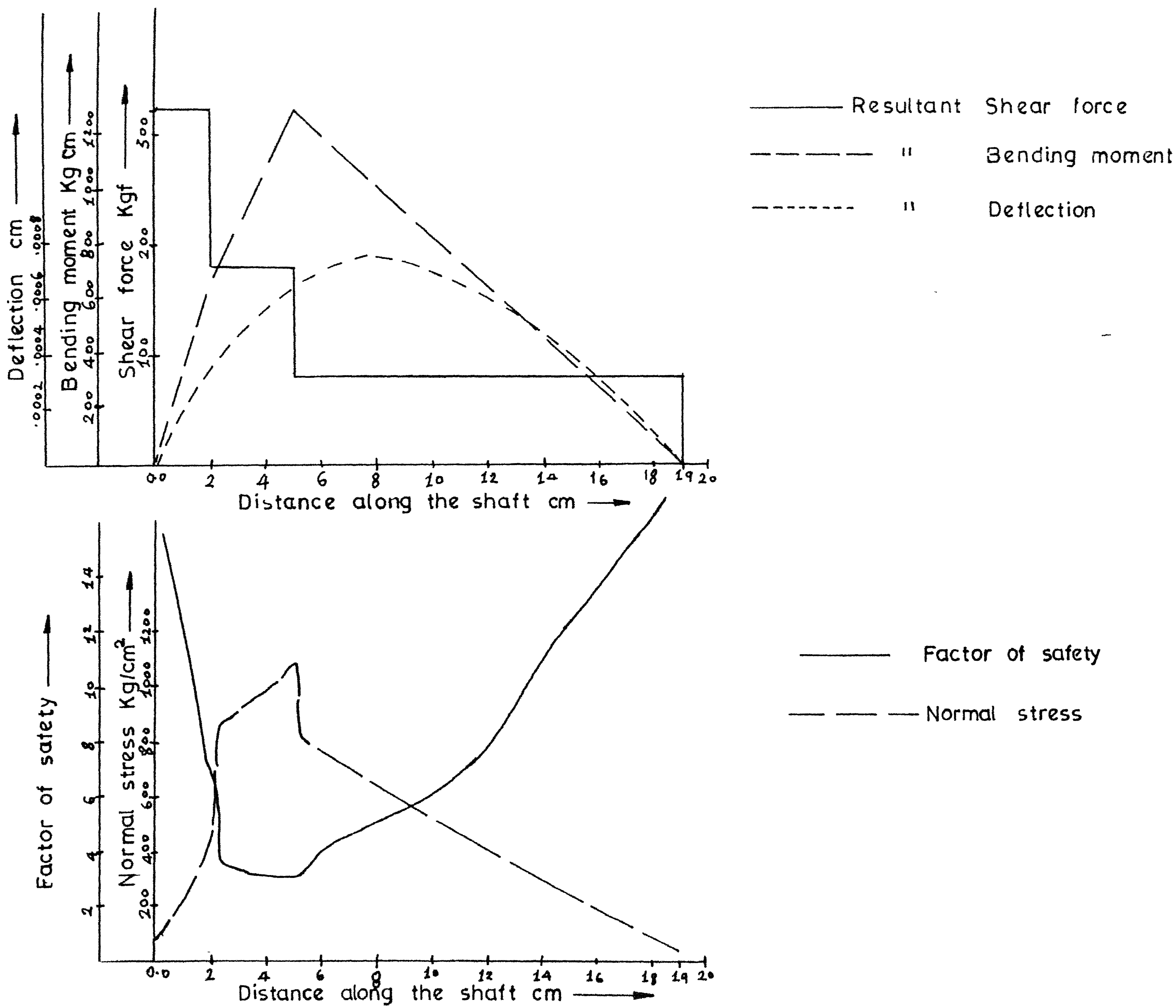


Fig. 5.5(b) Plots for Example 3 2nd gear arrangement

For shaft 2 in gear arrangement 1 the values of minimum factor of safety and maximum deflection are 1.749 and 0.0016 cm. respectively.
For shaft 2 in gear arrangement 1 the values of minimum factor of safety and maximum deflection are 2.?25 and 0.00075 cm respectively.

5.7 Example 4:

The aim of this problem is to design and analyse the 18 speed gear box shafts, for a given arrangement of gears. For a gear arrangement shown in Fig. (5.6) short and safe shafts are designed and analysed.
The inputs for this gear arrangement (taken from [15,18]) are as follows:

DO YOU WANT TO DESIGN AND ANALYSE GEAR BOX SHAFTS?
TYPE Y/N : Y
PLEASE TYPE IN THE MODULE OF THE GEARS IN CMS : 0.3 cm.
PLEASE TYPE IN THE PRESSURE ANGLE OF THE GEARS IN DEGREES : 20
PLEASE TYPE IN THE HORSE POWER OF THE INPUT MOTOR:6.711
PLEASE TYPE IN THE NUMBER OF SHAFTS IN THE GEAR BOX:5
PLEASE TYPE IN THE CODES FOR SLIDING AND NON-SLIDING SHAFTS : 1, 0, 1, C, 1
PLEASE TYPE IN THE ANGLE AT WHICH THE SECOND TO LAST SHAFTS ARE WITH THE FIRST SHAFT : 0.0, 0.0, 0.0, 0.0,0.0

PLEASE TYPE IN THE VALUES FOR DENSITY AND BREADTH OF GEARS : 0.0072, 3.0

PLEASE TYPE IN THE VALUES FOR YOUNG'S MODULUS AND SHEAR MODULUS : 21000000.0 8400000.0

PLEASE TYPE IN THE NUMBER OF GEARS ON EACH SHAFT: 3,6,4,3,2

PLEASE TYPE IN THE NUMBER OF TEETH ON EACH GEARS ON EACH SHAFT : 20, 34, 47, 56, 43, 27, 31, 33, 34, 37, 35, 32, 34, 20, 26, 35, 54, 47.

PLEASE TYPE IN THE POSITIONS OF THE GEARS FROM LEFT END BEARING ON INTERMEDIATE SHAFT: 2.0, 11.0, 20.0, 23.0, 32.0, 41.0 cms.

PLEASE TYPE IN THE POSITIONS OF THE RIGHT END BEARING: 46 cm.

PLEASE TYPE IN THE SEQUENCE OF THE GEARS FOR A PARTICULAR COMBINATION: 1,1,4,1,4,4,1,1

The diameters of the shafts for this arrangement are calculated and typed

PLEASE TYPE IN THE STANDARDISED DIAMETERS OF SHAFTS: 1.7, 2.5, 2.6, 2.8, 3.0

PLEASE TYPE IN THE STEPPED DIAMETERS ON THE NON-SLIDING SHAFTS: 2.5, 2.6, 2.8, 2.6, 2.5

PLEASE TYPE IN THE NUMBER OF KEYWAYS ON EACH SHAFT: 5

PLEASE TYPE IN THE POSITIONS OF THE KEYWAYS ON THE SHAFT: 0.5, 3.5, 18.5, 30.5, 42.5

PLEASE TYPE IN THE LENGTH OF EACH KEYWAY : 3.0, 6.0, 6.0, 3.0, 3.0

PLEASE TYPE IN THE STRESS CONCENTRATIONS AT KEYWAYS:1.2

PLEASE TYPE IN THE NOTCH SENSITIVITY FOR FATIGUE FAILURE:0.9

The diameters of all the shafts are calculated and they are standardised from the table typed on the terminal.

The diameter of the shafts are as follows,

The standardised diameter of shaft 1 is : 1.7 cm

2 is : 2.5 cm

3 is : 2.6 cm

4 is : 2.8 cm

5 is : 3.0 cm

Length of the gear box is 46 cm.

The results of the analysed intermediate shafts are tabulated in DATA files. The plots for the 2nd and 4th nonsliding shafts are shown in Fig. (5.7)a and Fig. (5.7)b.

Factor of safety for shaft 2 is 2.12 and for shaft 4 is 4.06

Maximum deflection for shaft 2 is 0.0094 cm and for shaft 4 is 0.003 cm.

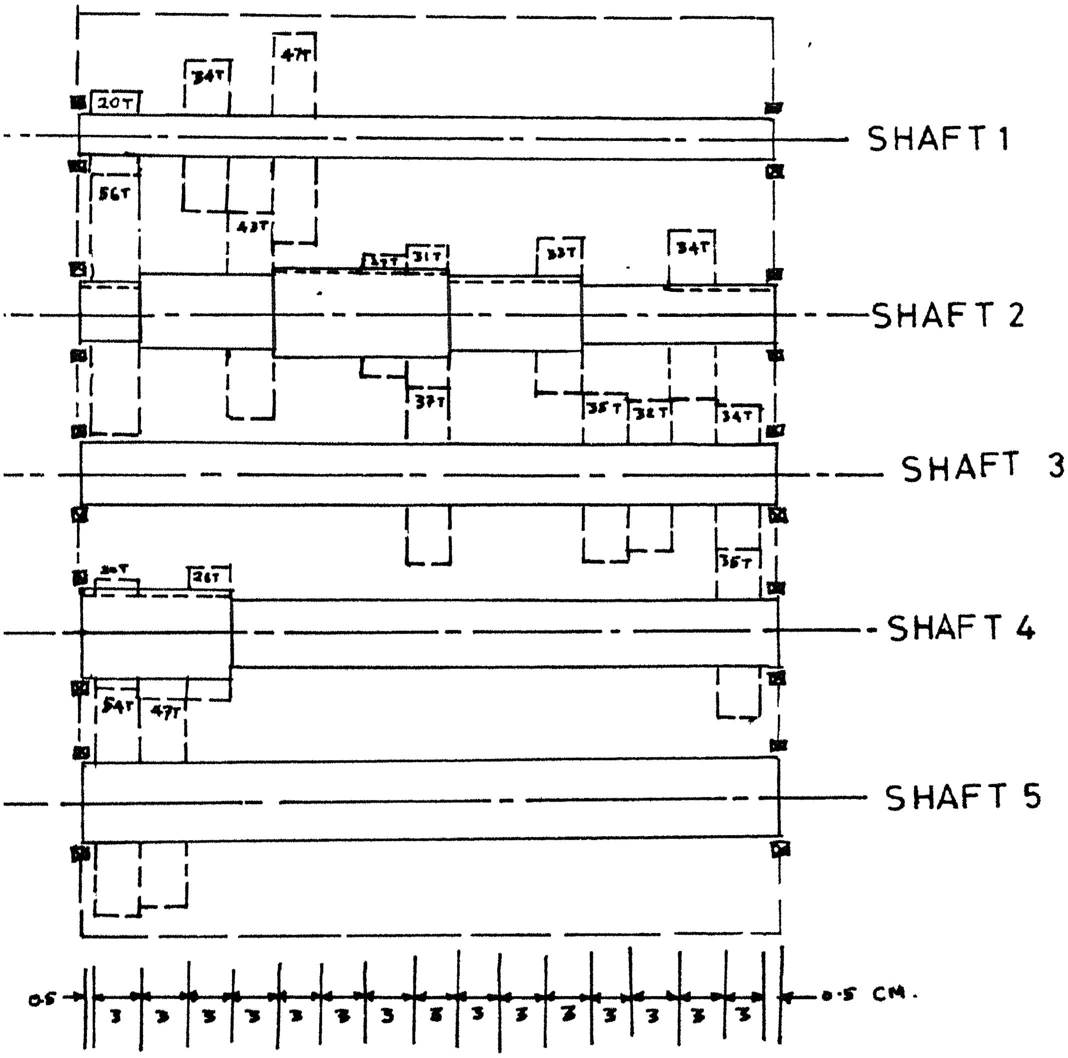


Fig. 5.6 Example 4, 18 Speed gear-box.

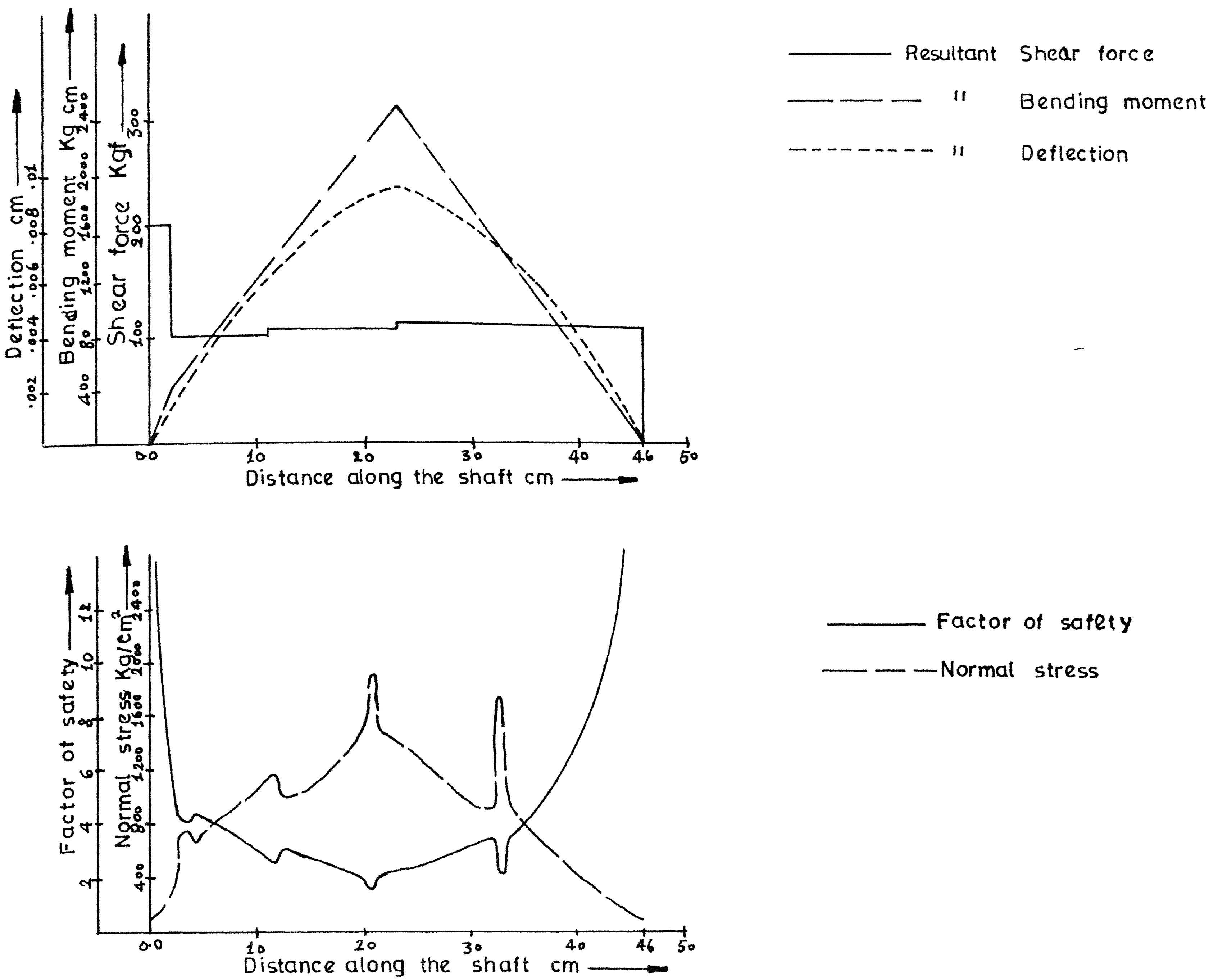


Fig. 5.7(a) Plots for 2nd shaft of 18 speed gear box.

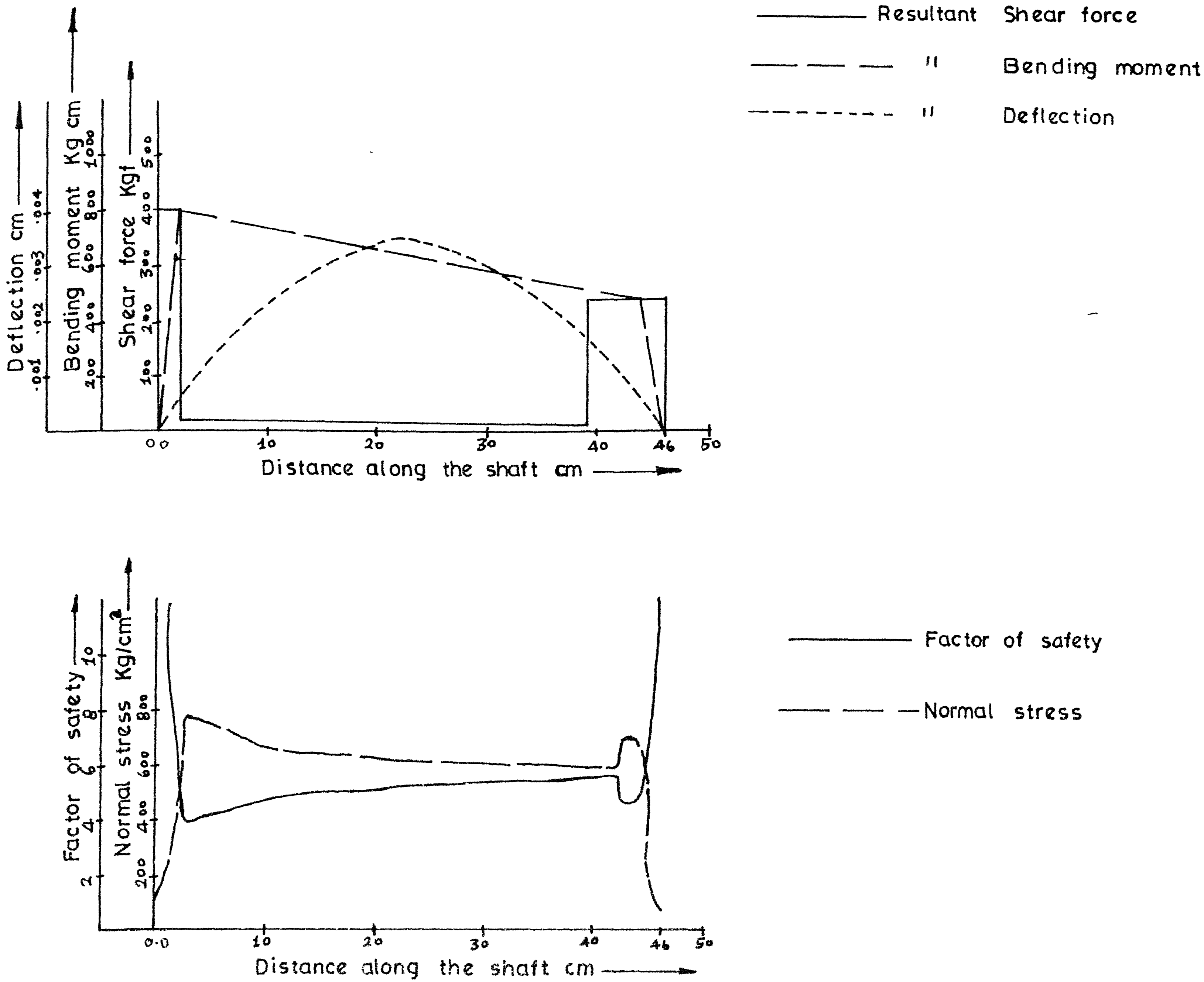


Fig 5.7(b) Plots for 4th shaft of 18 speed gear box.

CHAPTER-6

CONCLUSIONS

6.1 Technical Summary:

Computer aided design is a design process in which engineer uses the computer to design machine elements with the help of interactive computer programs. The present work is an attempt to develop an interactive design and analysis package for multi-speed gear box shafts. The program developed can also analyse shafts with circular cross section and general loadings.

The salient feature of the program developed is, that a designer can quickly explore all the alternative design strategies by analysing them and select the best of all, which would otherwise be time consuming. In case of gear box shafts, the designer can give different possible gear arrangements in gear box and t by analysing them through iterations, can select a best feasible shaft. For this purpose the program offers several iteractive features by which data can be modified and the entire design process can be repeated.

The purpose of shaft analysis is to determine the factor of safety of a shaft when given the geometry, material, and loading. In shaft design the two failure modes usually are yielding and fatigue, which are governed by the deflection and stress along the shaft. The transfer matrix approach works effectively in finding the deflection, slope, bending moment, and shear force at any location along the shaft. The differential equations used in the method are described in Chapter 2. Supports can be modelled as rigid or represented as springs that may be translational or rotational in nature. Stress concentrations are added to more accurately model the shaft. However, stress concentrations are assumed to occur only at critical portion and not over the distributed area. Fatigue failure is considered at critical portions. The distortion energy or the maximum shear stress failure theory along with the Soderberg equation gives a reasonable prediction of failure. This analysis procedure is very general to enable modelling of shafts with complex loading and geometry.

6.2 Recommendations for Further Work:

When required to design and analyse the gear box shafts, the designer has to give inputs to the program such as, the layout of the gears for each

arrangement. Keeping in mind the rules as to how to place them. These rules can be programed and thus one can get automatically the positions of the gears for each possible arrangement. Alternatively this layout of gear pairs and the location of support bearings can also be accomplished in an interactive graphical mode using a tablet or similar such input device.

When the sizes of the shafts are finalized, it is necessary to check that the operating speeds of the shafts do not coincide with the critical speeds of the shafts, this part can be included in the presently developed program.

The design of bearings at all support locations is also an important phase of the design process. Thus one can add in this work, the necessary design calcula - tions to select and size the appropriate type of bearing.

The failure theories, for brittle materials, can also be added for specific analysis.

Curve fitting algorithms can be added for the stress concentration plots, so that stress concentration factors can be considered directly in a computer program. To get the optimized size of the shaft, optimization methods can also be implemented.

REFERENCES

[1] Timoshenko, S.,
" Strength of Materials",
Robert E. Krieger Publishing Co. Inc. Huntington,
New York, 3rd edition, 1958.

[2] Boresi, Arthur P., Sidebottom, Omar M, Seely,
Fred B., and Smith, James O.,
" Advanced Mechanics of Materials",
John Wiley and Sons, Inc., New York,
3rd edition, New York, 1978.

[3] Sen, G.C. and Bhattacharyya, A.
" Principles of Mache Tools",
New Central Book Agency, India, 1975.

[4] Central Machine Tool Institute Bangalore
" Machine Tool Design Handbook",
Tata McGraw-Hill Publishing Co. Ltd.,
New Delhi, 1982.

[5] White, G.,
" Four Speed Gearbox with Six Gears",
International Journal of Machine Tool Design
and Research, Vol. 3, 1964.

[6] Pestel, Edward C., and Leckie, Frederick, A.,
" Matrix Methods in Elastomechanics",
McGraw-Hill Book Co. Inc., New York
New York, 1963.

[7] Pilkey, Walter, D., and Chang, Pin Yu,
" Modern Formulas for Statics and Dynamics",
McGraw-Hill Book Co. Inc., New York
New York, 1978.

[8] Spotts, M.F.,
" Design of Machine Elements",
Prentice-Hall Inc. Englewood Cliffs, New Jersey,
5th edition, 1978.

[9] Roark, Raymond, J., and Young, Warren, C.,
" Formulas for Stress and Strain",
McGraw-Hill Book Co. Inc., New York, 5th edition,
New York, 1975.

[10] Shigley, J.E.,
" Mechanical Engineering Design",
McGraw-Hill Book Co. Inc.,New York,
3rd edition, New York, 1977.

[11] Hopkins, Bruce, R.,
" Design Analysis of Shafts and Beams",
McGraw-Hill Book Co. Inc., New York,
New York, 1970.

[12] Pilkey Walter, D., and Pilkey, Orvin, H.,
"Mechanics of Solids",
Quantum Publishers, Inc., New York,
New York, 1974.

[13] Peterson, R.E.,
" Stress Concentration Factor",
John Wiley and Sons, Inc., New York,
New York, 1974.

[14] Pilkey, W.D., Chu, F.H.,
" A Direct Integration Technique for the
Transient Analysis of Rotating Shafts",
Trans. ASME, Vol. 104, April 1982, pp 384-388.

[15] Gupta, A.,
" Cumputer-Aided Interactive, Graphical Design
of Multispeed Machine Tool Gearbox",
M. Tech. thesis, I.I.T., Kanpur, April, 1982.

[16] Lingaiah, K.,
" Machine Design Data Handbook",
Volume 1, Suma Publishers Bangalore, 1973.

[17] Besant, C.B.,
" Computer Aided Design and Manufacture",
John Wiley and Sons, New York,
New York, 1980.

[18] Dhande, S.G., Gupta, A.,
" Computer Aided Interactive Graphical Design
of Multispeed Gearboxes",
ASME, 83-DET-49, Nov. 1983.

[19] Newman, W.M., and Sproull, R.F.,
" Principles of Interactive Computer Graphics",
McGraw-Hill, Int. Student Edition, 2nd edition,
1979.

[20] Mudur, S.P.,
" General Purpose Graphics System User's Manual",
NCSDCT Publication TIFR Bombay, December, 1975.

APPENDIX-A

DERIVATION OF TRANSFER MATRICES

(A.1) Introduction:

There are number of ways to obtain the transfer matrices from the equations of motion. The most direct of available methods is simply to rearrange the arbitrary constants of integration of solution in terms of the initial parameters [7,12]. Here only field matrix development is discussed.

The fundamental equations of motion in first order form for the bending of a Timoshenko beam are,

$$\frac{\partial y}{\partial x} = -\theta + \frac{V}{GA_s} \tag{A.1}$$

$$\frac{\partial \theta}{\partial x} = \frac{M}{EI} + \frac{M_T}{EI} \tag{A.2}$$

$$\frac{\partial M}{\partial x} = V + (K^{*}-P)\,\theta - d\,\rho\, r_y^2 \frac{\partial^2 \theta}{\partial t^2} - c(x,t) \quad \ldots \tag{A.3}$$

$$\frac{\partial V}{\partial x} = Ky + \rho \frac{\partial^2 y}{\partial t^2} - W\,(x,t) \tag{A.4}$$

These partial differential equations of motion for beams; in the case of steady-state motion, the loadings and responses (deflection, slope, bending moment, and shear force) all vary harmonically and reduces to ordinary differential equations as,

$$\frac{dy}{dx} = - \theta + \frac{V}{GA_s} \qquad (A.5)$$

$$\frac{d\theta}{dx} = \frac{M}{EI} + \frac{M_T}{EI} \qquad (A.6)$$

$$\frac{dM}{dx} = V + (K^* - P - \rho\, r^2 . \omega^2)\theta - c \qquad (A.7)$$

$$\frac{dV}{dx} = (K - \rho\omega^2)\, y - W \qquad (A.8)$$

These equations apply to the free vibration of a beam if the applied loadings C, W, M_T are set equal to zero. Also, they are appropriate for static responses if ω is set equal to zero.

(A.2) Laplace Transformation:

In matrix notation Equations (A.5) to (A.3) can be written as

$$\frac{dS}{dx} = AS + f \qquad (A.9)$$

where,

$$S = \begin{bmatrix} y \\ \theta \\ M \\ V \end{bmatrix}, \quad f = \begin{bmatrix} 0 \\ M_{T1}/EI \\ -(c_1 + \frac{\Delta c}{\Delta l}) \\ -(W_1 + \frac{\Delta W}{\Delta l}) \end{bmatrix}$$

and

$$A = \begin{bmatrix} 0 & -1 & 0 & 1/GA_s \\ 0 & 0 & 1/EI & 0 \\ 0 & K^* - P - \rho r^2 \omega^2 & 0 & 1 \\ K - \rho\omega^2 & 0 & 0 & 0 \end{bmatrix} \quad ..(A-10)$$

The Laplace transform of Equation (A.9) is written as

$$s\,S(s) = S(0) + AS(s) + f(s) \qquad (A.11)$$

then

$$S(s) = (IS - A)^{-1}\,S(0) + (IS - A)^{-1}\,f\,(s)$$
$$\ldots\ldots \qquad (A.12)$$

where I is a unit diagonal matrix. The inverse Laplace transform is

$$S(x) = L^{-1}\,[(IS - A)^{-1}]S(0) + {}^{-1}[(IS - A)^{-1}f(s)]$$
$$\ldots \qquad (A.13)$$

The term $L^{-1}\,[(IS - A)^{-1}]$ is the 4 x 4 transfer matrix, and the second term in equation (A.13) is the fifth column of the extended transfer matrix.

Now to find a field matrix for a Euler Bernoulli beam with shear deformation. Choose the case of a beam considering shear deformation with ρ, K, K^*, P, $\Delta W/\Delta l$, c_1, $\Delta c/\Delta l$, M_T, equal to zero, then

$$A = \begin{bmatrix} 0 & -1 & 0 & 1/GAs \\ 0 & 0 & 1/EI & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -W_1 \end{bmatrix} \qquad (A.14)$$

$$(IS-A)^{-1} = \begin{bmatrix} \frac{1}{s} & -\frac{1}{s^2} & -\frac{1}{s^3EI} & -\frac{1}{s^4EI} + \frac{1}{s^2GA_s} \\ 0 & \frac{1}{s} & \frac{1}{s^2EI} & \frac{1}{s^3EI} \\ 0 & 0 & \frac{1}{s} & \frac{1}{s^2} \\ 0 & 0 & 0 & \frac{1}{s} \end{bmatrix} \quad \text{(A.15)}$$

$$L^{-1}[(IS-A)^{-1}] = \begin{bmatrix} 1 & -x & -\frac{x^2}{2EI} & -\frac{x^3}{6EI} + \frac{x}{GA_s} \\ 0 & 1 & x/EI & x^2/2EI \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(A.16)}$$

$$(IS-A)^{-1}\ f\ (s) = \begin{bmatrix} W_1/s^5EI - W_1/s^3GA_s \\ -W_1/s^4EI \\ -W_1/s^3 \\ -W_1/s^2 \end{bmatrix} \quad \text{(A.17)}$$

$$L^{-1}[(IS-A)^{-1}f(s)] = \begin{bmatrix} \frac{W_1\ x^4}{24EI} - \frac{W_1x^2}{2GA_s} \\ -\frac{W_1\ x^3}{6EI} \\ -\frac{W_1\ x^2}{2} \\ -W_1x \end{bmatrix} \quad \text{(A.18)}$$

Combining equations (A.13), (A.16), and (A.18) in the extended transfer matrix form

$$\begin{bmatrix} y \\ \theta \\ M \\ V \\ 1 \end{bmatrix}_x = \begin{bmatrix} 1 & -x & -\frac{x^2}{2EI} & -\frac{x^3}{6EI} + \frac{x}{GA_s} & \frac{W_1x^4}{24EI} - \frac{W_1x^2}{2GA_s} \\ 0 & 1 & \frac{x}{EI} & \frac{x^2}{2EI} & -\frac{W_1x^3}{6\ EI} \\ 0 & 0 & 1 & x & -\frac{W_1x^2}{2} \\ 0 & 0 & 0 & 1 & -W_1x \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ \theta \\ M \\ V \\ 1 \end{bmatrix}_0 \quad \ldots \quad (A.19)$$

Similarly it is possible to derive the point matrix by this Laplace transformation.

APPENDIX-B

EXAMPLE OF TRANSFER MATRIX METHOD

(B.1) Introduction:

The transfer matrix method is capable of modelling complicated beams or shafts. An example of this method's solution procedure, to a statically indeterminate beam with an in-span rigid support is given in the following section below. The procedure for incorporating in-span rigid supports is complex, but straightforward in application.

(B.2) Transfer Matrix Approach:

The purpose of this section is to give a detailed example with sequential steps for the transfer matrix method. The example given in Fig. (B.1) is a cantilever beam with a rigid in-span support and a point load at the overhung end. The objective here is to determine the deflection, slope, bending moment and shear force at $x = 0$, $x = L_1$ and $x = L$. The major objective is to determine the state vector at left end ($x = 0$), because then the deflection, slope, bending moment, and shear force can be calculated at any point along the beam. First, the beam must be modelled in terms of sections that connect abrupt geometric changes or point

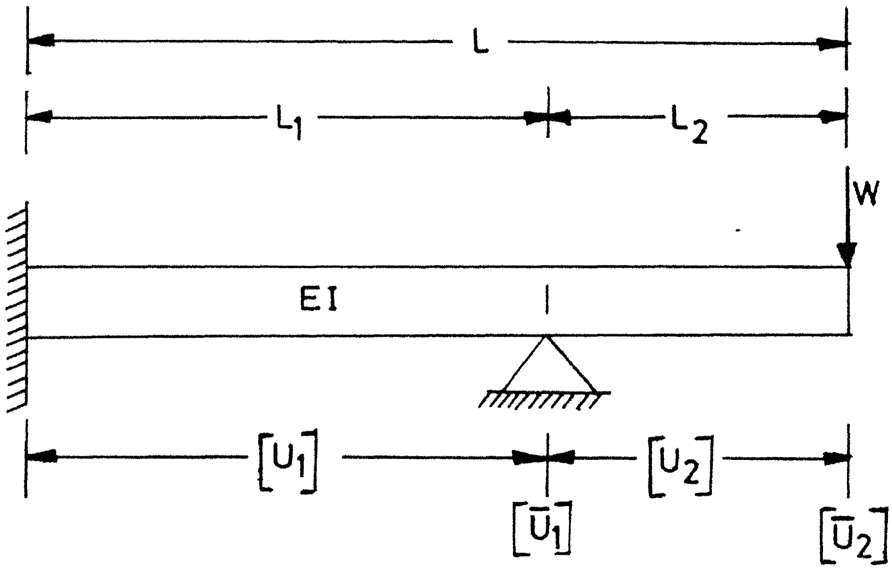


Fig. B.1 Cantilever beam with an in-span rigid support.

occurrences. Two sections exist for the example $x = 0$ to $x = L$ and from $x = L_1$ to $x = L$, since a rigid in-span support occurs at $x = L_1$, as shown in Fig. (B.1). Assuming, no geometric changes occur along the beam and all the material properties are known, the transfer matrices along the beam can be formed. To apply the boundary conditions, the global matrix must be formed from the following equation,

$$\{S\}_{x=L} = [\bar{U}_2]\,[U_2]\,[\bar{U}_1]\,[U_1]\,\{S\}_o = [U]\,\{S\}_o \quad \ldots\ldots \quad (B.1)$$

In this equation $[U_1]$ and $[U_2]$ are field matrices and $[\bar{U}_1]$ is point matrix for rigid in-span support at $x = L_1$ and $[\bar{U}_2]$ is point matrix for the point load W.

The field matrix $[U_1]$ is for $x = o$ to $x = L_1$ and form eq. (2.19) is equal to

$$[U_1] = \begin{bmatrix} 1 & -L_1 & -L_1^2/2EI & -L_1^3/6EI + L/GA_s & F_Y \\ 0 & 1 & L_1/EI & L_1^2/2EI & F_\theta \\ 0 & 0 & 1 & L_1 & F_M \\ 0 & 0 & 0 & 1 & F_V \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots \quad (B.2)$$

Since shear deformation is very small compared with the term $-L_1^3/6EI$, thus is neglected in present example.

The terms in equation (B.2) F_y, F_θ, F_M and F_V are equal to zero since the beam is not loaded with uniformly distributed or linearly distributed loads. Thus the field matrix $[U_1]$ for x = o to $x = L_1$ is

$$[U_1] = \begin{bmatrix} 1 & -L_1 & -L_1^2/2EI & -L_1^3/6EI & 0 \\ 0 & 1 & L_1/EI & L_1^2/2EI & 0 \\ 0 & 0 & 1 & L_1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (B.3)$$

Similarly the field matrix $[U_2]$ for $x = L_1$ to $x = L$ can be given as

$$[U_2] = \begin{bmatrix} 1 & -L_2 & -L_2^2/2EI & -L_2^3/6EI & 0 \\ 0 & 1 & L_2/EI & L_2^2/2EI & 0 \\ 0 & 0 & 1 & L_2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (B.4)$$

$[\bar{U}_2]$ is the point matrix for the concentrated load at the end of the beam and is given by using Table (2.2) as,

$$[\,\bar{U}_2] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -W \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (B.5)$$

To determine $[\,\bar{U}_1]$, which is a point matrix for the rigid support, the subglobal matrix from $x = L_1$ to $x = L$ must be calculated. The subglobal matrix is equal to $[\,\bar{U}_2]$ times $[\,U_2]$ and becomes

$$[\bar{U}_2][U_2] = \begin{bmatrix} 1 & -L_2 & -L_2^2/2EI & -L_2^3/6EI & 0 \\ 0 & 1 & L_2/EI & L_2^2/2EI & 0 \\ 0 & 0 & 1 & L_2 & 0 \\ 0 & 0 & 0 & 1 & -W \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (B.6)$$

After obtaining the subglobal matrix, one of the boundary conditions at the right end is applied to calculate the point matrix $[\bar{U}_1]$ for in-span support in Table (2.3). Since beam is free at the right end the boundary conditions at right end are that the bending moment and shear force are equal to zero ($M_{x=l} = 0$ and $V_{x=l} = 0$). Thus letting $s_m = V$ in Table (2.3), $[\,\bar{U}_1]$ can be found to be as

$$[\bar{U}_1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & W \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(B.7)}$$

By multiplying all the transfer matrices from left to right, the global matrix is equal to

$$[U] = [\bar{U}_2]\ [U_2']\ [\bar{U}_1]\ [U_1]$$

which becomes as

$$[U] = \begin{bmatrix} 1 & -L_1-L_2 & \dfrac{-L_1^2}{2EI} - \dfrac{L_1L_2}{EI} - \dfrac{L_2^2}{2EI} & \dfrac{-L_1^3}{6EI} - \dfrac{L_1^2L_2}{2EI} - \dfrac{L_1L_2^2}{2EI} & \dfrac{-L_2^3}{6EI}\,W \\ 0 & 1 & \dfrac{L_1}{EI} + \dfrac{L_2}{EI} & \dfrac{L^2}{2EI} + \dfrac{L_1L_2}{EI} & \dfrac{L_2^2}{2EI}\,W \\ 0 & 0 & 1 & L_1 & L_2\,W \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\ldots\ldots \qquad \text{(B.9)}$$

Now, the remaining boundary condition at the right end ($M_{x=L} = 0$) and the boundary condition at the rigid in-span support are applied as outlined in Section (2.8).

The following equations are formed

$$y_{x=L_1} = 0 = U_{yM} M_o + U_{yV} V_o + F_y \quad (B.10)$$

$$M_{x=L} = 0 = U_{MM} M_o + U_{MV} V_o + F_M \quad (B.11)$$

The terms, U_{yM} , U_{yV} and F_y , come from the subglobal matrix from $x = 0$ to $x = L_1$, and in this case this subglobal matrix is $[U_1]$, substituting the desired terms from the subglobal matrix, equation (B.10) becomes

$$0 = M_o \left(- L_1^2/2EI\right) + V_o \left(- L_1^3/6EI\right) \quad (B.12)$$

The elements from the global matrix U_{MM}, U_{MV} and F_M , are substituted into equation (B.11), which leads to

$$0 = M_o + V_o L_1 + L_2 W \quad (B.13)$$

After simultaneously solving equations (B.12) and (B.13) for the unknown state vectors, M_o and V_o become

$$V_o = - 3L_2 W/2L_1 \quad (B.14)$$

$$M_o = L_2 W/2 \quad (B.15)$$

And since the boundary conditions at left end which is fixed, are $y_o = 0$ and $\theta_o = 0$, the state vector at left end $x = o$ can be given by using above equations (B.14) and (B.15) and boundary conditions, as

$$\left\{ S \right\}_{x=o} = \begin{bmatrix} 0 \\ 0 \\ L_2 W/2 \\ - 3L_2 W/2L_1 \end{bmatrix} \quad (B.16)$$

The deflection, slope, bending moment, and shear force are calculated at the locations of interest by multiplying the required matrices together with the state vector at the left end. For example, to find the deflection, slope, bending moment, and shear force at just to the left of $x = L_1$, the following equation is developed.

$$\{S\}_{x=L_1} = [U_1] \{S\}_{x=o} \tag{B.17}$$

or

$$\begin{bmatrix} y \\ \theta \\ M \\ V \\ 1 \end{bmatrix}_{x=L_1} = \begin{bmatrix} 1 & -L_1 & -L_1^2/2EI & -L_1^3/6EI & 0 \\ 0 & 1 & L_1/EI & L_1^2/2EI & 0 \\ 0 & 0 & 1 & L_1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ L_2W/2 \\ -3L_2W/2L_1 \\ 1 \end{bmatrix}$$

$$\ldots \tag{B.18}$$

This gives

$$\{S\}_{x=L_1} = \begin{bmatrix} 0 \\ -W\, L_1L_2/4EI \\ -\, W\, L_2 \\ -3L_2\, W/2L_1 \\ 0 \end{bmatrix} \tag{B.19}$$

The other location of interest is at x = L. The state vector at this point is found by using eauations,

$$\{S\}_{x=L_1} = [\bar{U}_2][U_2][\bar{U}_1][U_1]\{S\}_o \qquad (B.20)$$

or

$$\{S\}_{x=L} = [\bar{U}_2][U_2]([\bar{U}_1][U_1]\{S\}_{x=L_1} \qquad (B.21)$$

by using equation (B.21), the successive state vectors can be calculated just by using the previous state vectors and proper transfer matrices in between them.

Thus the state vector at x = L is given as

$$\{S\}_{x=L} = \begin{bmatrix} W L_1 L_2^2/4EI + L_2^3 W/3EI \\ -W L_1 L_2/4EI - L_2^2 W/2EI \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (B.22)$$

The in between state vectors can be computed by adjusting the length coordinates in the field matrix for the perticular sections and by properly arranging the transfer matrices.